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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

EC3551 - TRANSMISSION LINES AND RF SYSTEMS
(REGULATION – 2021)

YEAR: III **SEM: V**

UNIT II – HIGH FREQUENCY TRANSMISSION LINES

PART – A

1. STATE THE ASSUMPTIONS FOR THE ANALYSES OF THE PERFORMANCE OF THE RADIO FREQUENCY LINE?

* Due to the skin effect, the currents are assumed to flow on the surface of the conductor. The internal inductance is zero
* The resistance R increases with $\sqrt{f}$ while inductance L increases with f. Hence $\omega L >> R$
* The leakage conductance G is zero.

2. STATE THE PARAMETERS OF A OPEN WIRE LINE AT A HIGH FREQUENCY?

i) $L = 9.21 \times 10^{-7} \log \frac{d}{a}$ henrys/m ii) $C = \dfrac{12.07}{\log \frac{d}{a}}$ $\mu\mu f/m$

iii) $R_{dc} = \dfrac{k}{\pi a^2}$ iv) $R_{ac} = \dfrac{k}{2\pi a \delta}$ $\left[\because \delta = \dfrac{0.0664}{\sqrt{f}}\right]$

3. WHAT ARE CONSTANTS FOR ZERO DISSIPATION LINE?

$\alpha = 0$

$\beta = \omega\sqrt{LC}$

$Z_0 = R_0 = \sqrt{L/C}$

$V = \dfrac{1}{\sqrt{LC}}$

4. WHAT ARE NODES AND ANTINODES ON A LINE?

The points along the line where magnitude of voltage or current is zero are called nodes while the points along the lines where magnitude of voltage or current first maximum are called antinodes or loops.

5. WHAT IS STANDING WAVE RATIO? STATE THE RELATION BETWEEN STANDING WAVE RATIO 'S' AND REFLECTION COEFFICIENT k?

* The ratio of the maximum to minimum magnitudes of Voltages or currents on a line having standing waves is called standing wave ratio

$$S = \frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

* The relationship is given as

$$S = \frac{1+|k|}{1-|k|} \quad \text{OR} \quad |k| = \frac{S-1}{S+1}$$

6. SPECIFY THE RANGE OF VALUES OF STANDING WAVE RATIO AND REFLECTION COEFFICIENT?

* The range of values of standing wave ratio is theoretically 1 to infinity

* The range of values reflection coefficient is theoretically 0 to 1

7. WHAT ARE STANDING WAVES? WHY THEY CALLED SO?

If the transmission is not terminated in its characteristic impedance, the Voltage at any point in a line is the sum of incident and reflected Voltage. This resultant Voltage stand still on the line having fixed maximum and minimum positions called standing waves.

8. WHAT IS SMOOTH LINE?

* A smooth line is the one which terminated in its characteristics impedance and no reflection occurs in smooth line

9. WHEN DOES REFLECTION TAKES PLACE IN A TRANSMISSION LINE?

* Reflection occurs because of the following cases:

1. when the load end is open circuited
2. when the load end is short circuited
3. when the line is not terminated in its characteristic impedance

10. WRITE THE EXPRESSION FOR POWER FLOW IN A TRANSMISSION LINE?

$$P = \frac{|E_{max}||E_{min}|}{R_o}$$

$$= |I_{max}||I_{min}| R_o$$

11. WRITE EQUATION FOR CHARACTERISTICS IMPEDANCE AND PROPAGATION CONSTANT OF A DISSIPATION LESS LINE?

$Z_o = R_o = \sqrt{L/C}$ assumption:

$\gamma = 0 + j\omega\sqrt{LC}$ $j\omega L >> R$

$\alpha = 0$

$\beta = \omega\sqrt{LC}$ $j\omega C >> G$

12. DEFINE STANDING WAVE RATIO?

The ratio of the maximum to minimum magnitudes of voltages or current on a line having standing wave is called SWR

$$S = \frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

13. WRITE THE EQUATION FOR REFLECTION COEFFICIENT IN TERMS OF MAXIMUM AND MINIMUM VOLTAGES?

$$K = \frac{|V_{max}| - |V_{min}|}{|V_{max}| + |V_{min}|}$$

14. WRITE EQUATIONS FOR INDUCTANCE AND CAPACITANCE OF AN OPEN WIRE LINE AT HIGH FREQUENCIES?

$$L = 4 * 10^{-7} \ln (d/a) \text{ henry/m}$$

$$C = 27.7 / \ln (d/a) \ \mu\mu F/m$$

where d = distance between conductors

a = radius of each conductors

15. WRITE EQUATIONS FOR INDUCTANCE AND CAPACITANCE OF A COAXIAL LINE AT HIGH FREQUENCIES?

$$L = 2 * 10^{-7} \ln (b/a) \text{ henrys/m}$$

$$C = 55.5 / \ln (b/a) \ \mu\mu F/m$$

where a = radius of inner conductor.

b = inner radius of outer conductor.

PART-B

(1) EXPLAIN THE PARAMETERS OF OPEN WIRE AND COAXIAL CABLE AT HIGH FREQUENCY [Nov/Dec 2014] [Nov/Dec 2011]

(i) PARAMETERS OF THE OPEN WIRE LINE AT HIGH FREQUENCY

* At high frequency, the current is considered as flowing on the surface of the conductor in a skin of very small depth. The internal flux & internal inductance are then reduced nearly to zero.

INDUCTANCE

Inductance of an open wire line is

$$L = \frac{\mu_0}{\pi} \ln \frac{d}{a}$$

$\mu_0 \rightarrow$ permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}$$

$$L = \frac{4\pi \times 10^{-7}}{\pi} \ln \frac{d}{a} \quad \text{henrys/m}$$

$$= 4\pi \times 10^{-7} \ln \frac{d}{a} \qquad \ln = 2.303 \log$$

$$= 4\pi \times 10^{-7} \times 2.303 \log \frac{d}{a}$$

$$\boxed{L = 9.21 \times 10^{-7} \log \frac{d}{a} \quad \text{henrys/m}}$$

CAPACITANCE

The value of capacitance is not affected by skin effect or frequency

Capacitance of an open wire line is

$$C = \frac{\pi \varepsilon_0}{\ln \frac{d}{a}} \quad \text{farads/m}$$

$\varepsilon_0 \rightarrow$ permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12} = \frac{10^{-9}}{36\pi}$$

$$C = \frac{\pi \times 8.85 \times 10^{-12}}{\ln \frac{d}{a}}$$

$$= \frac{27.7 \times 10^{-12}}{\ln \frac{d}{a}} \; f/m$$

$$= \frac{27.7}{\ln \frac{d}{a}} \; \mu\mu f/m$$

$$= \frac{27.7}{2.303 \log \frac{d}{a}} \; \mu\mu f/m$$

$$\boxed{C = \frac{12.07}{\log \frac{d}{a}} \; \mu\mu f/m}$$

SKIN DEPTH

In the case of skin effect, the current flows over the surface of the conductor in a thin layer, so the effective cross section of the conductor will be reduced and an increase in resistance of the conductor.

The effective thickness of the surface layer of currents is given by

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \; \text{metres}}$$

For copper conductors $\mu = \mu_0 = 4\pi \times 10^{-7}$ henrys/metre

$\sigma = 5.75 \times 10^{7}$ mhos/metre at $20°c$

$$\delta = \frac{1}{\sqrt{\pi f \times 4\pi \times 10^{-7} \times 5.75 \times 10^{7}}}$$

$$\boxed{\delta = \frac{0.0664}{\sqrt{f}}}$$

Resistance

* The resistance of a round conductor of radius a meters to direct current is inversely proportional to the area

$$R_{dc} \propto \frac{1}{\pi a^2}$$

$$\boxed{R_{dc} = \frac{k}{\pi a^2}}$$

k → Constant of proportionality

* The resistance of a round conductor with alternating current flowing in a skin of thickness δ is

$$R_{ac} \propto \frac{1}{2\pi a \delta}$$

$$\boxed{R_{ac} = \frac{k}{2\pi a \delta}}$$

* The ratio of resistance to alternating current to resistance to direct current is

$$R = \frac{R_{ac}}{R_{dc}} = \frac{k\pi a^2}{2\pi a \delta k}$$

$$= \frac{a}{2\delta}$$

$$\boxed{\frac{R_{ac}}{R_{dc}} = \frac{a}{2}\sqrt{\pi f \mu \sigma}}$$

For Copper line,

$$R = \frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{f}}{2 \times 0.0664}$$

$$= 7.53\sqrt{f} \times a = 7.53a\sqrt{f}$$

where f in cycles per Second

a in meters

The above equation shows that resistance increases with increase in frequency. Also resistance increases for large radius Conductors

Thus for an open wire line at high frequencies the line parameters are found to be

$$R = \frac{a\sqrt{\pi f \mu \sigma}}{2} \text{ ohms/m}$$

$$L = 9.21 \times 10^{-7} \log \frac{d}{a} \text{ henrys/m}$$

$$C = \frac{12.07}{\log \frac{d}{a}} \text{ } \mu\mu f/m$$

(ii) PARAMETERS OF THE COAXIAL LINE AT HIGH FREQUENCY

At high frequencies in the Coaxial line, the current flows on the outer Surface of the inner Conductor and the inner Surface of the Outer Conductor because of the Skin effect. This phenomenon eliminates flux linkage due to internal Conductor flux

INDUCTANCE

The inductance of the Coaxial line is

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = \frac{4\pi \times 10^{-7}}{2\pi} \ln \frac{b}{a} \text{ henrys/m}$$

$$\boxed{L = 2 \times 10^{-7} \ln \frac{b}{a} \text{ henrys/m}}$$

$$L = 2\times10^{-7} \times 2.303 \log \frac{b}{a} \text{ henrys/m}$$

$$\boxed{L = 4.606 \times 10^{-7} \log \frac{b}{a} \text{ henrys/m}}$$

CAPACITANCE

The capacitance of the coaxial line is not affected by frequency

The capacitance of the coaxial line is

$$C = \frac{2\pi\varepsilon}{\ln \frac{b}{a}} \text{ farads/m}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$C = \frac{2\pi\varepsilon_0 \varepsilon_r}{\ln \frac{b}{a}} \text{ farads/m}$$

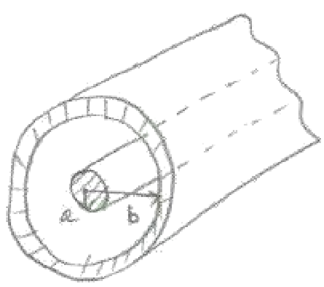
$$C = \frac{2\pi \times 8.85 \times 10^{-12} \times \varepsilon_r}{\ln \frac{b}{a}}$$

$$\boxed{C = \frac{55.6 \times 10^{-12}\, \varepsilon_r}{\ln \frac{b}{a}} \text{ f/m}}$$

$$C = \frac{55.6\, \varepsilon_r}{2.303 \log \frac{b}{a}} \ \mu\mu\text{ f/m}$$

$$\boxed{C = \frac{24.14\, \varepsilon_r}{\log \frac{b}{a}} \ \mu\mu\text{ f/m}}$$

RESISTANCE



The resistance of a Copper Coaxial line is.

$$\boxed{R_{ac} = 4.16 \times 10^{-8}\sqrt{f}\left(\frac{1}{b}+\frac{1}{a}\right) \text{ ohms/m}}$$

where $a \rightarrow$ Outer radius of the inner Conductor

$b \rightarrow$ Inner radius of the Outer Conductor.

CONDUCTANCE

* If air is dielectric, shunt losses are zero
* If dielectric is of Said type, the Conductance losses have to be Considered, at high frequencies

The quality of the insulating material (dielectric) may be measured interms of the power factor of the material

The shunt admittance is given by

$$y = g + j\omega c$$

Power factor can be expressed from the Susceptance triangle as the Cosine of θ

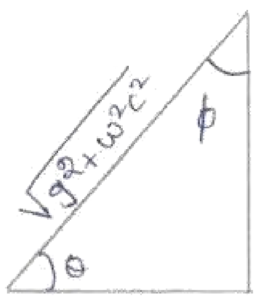


Fig: Loss triangle for dielectric

$$\therefore Pf = \cos(\theta)$$

$$= \frac{g}{\sqrt{g^2+\omega^2c^2}}$$

For a good insulating material, Conductance is Very Small (ie) $g << \omega c$

$$\therefore Pf = \frac{g}{\sqrt{\omega^2c^2}}$$

$$= \frac{g}{\omega c}$$

$$\boxed{g = \omega c \times pf}$$

The quality of dielectric can also be expressed interms of dissipation factor

Dissipation factor is defind as the ratio of energy dissipates to energy stored in the dielectric per cycle and is proportional to tangent of the angle ϕ.

Dissipation factor $= \tan(\phi)$

For good dielectrics with small power factor angle, the dissipation factor and power factor are equal in magnitude

Q (i) DERIVE VOLTAGE AND CURRENT ON THE DISSIPATION LESS LINE (10)

VOLTAGE AND CURRENTS ON THE DISSIPATIONLESS LINE

The Voltage at any point distant S units from the receiving end of a transmission line is

$$E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma s} + ke^{-\gamma s}\right]$$

W.k.T $\gamma = \alpha + j\beta$

$$\therefore E = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[e^{\alpha s + j\beta s} + ke^{-\alpha s - j\beta s}\right]$$

For the line of zero dissipation, the attenuation constant α is zero and Z_0 and R_0.

$$\therefore E = \frac{E_R(Z_R+R_0)}{2Z_R}\left(e^{j\beta s} + ke^{-j\beta s}\right)$$

In the above equation

* The term associated with $e^{j\beta s}$ is the incident wave progressing from the source towards the load

* The term associated with $e^{-j\beta s}$ is the reflected wave progressing from the load towards the source

* The magnitude of the reflected wave is dependent on the value of k, the reflection coefficient

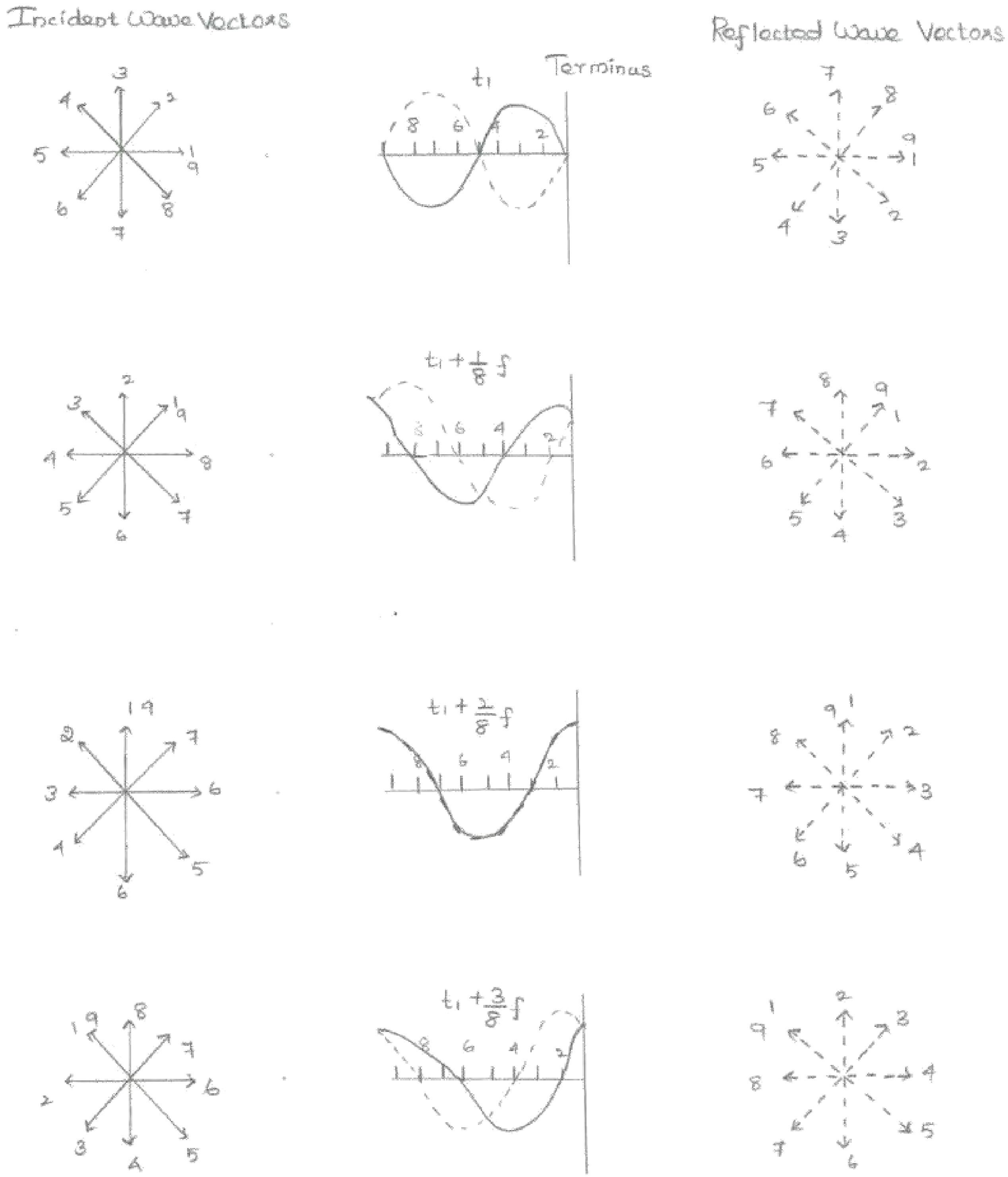
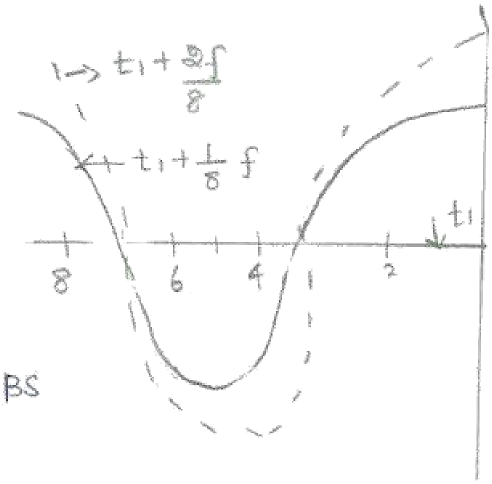


Fig: Incident and reflected voltage-wave phasors and values along the dissipationless line for successive instants of time, for an open-circuited line

* In the absence of attenuation, the rotating Vectors of both the incident and reflected waves remains constants in magnitude at all points on the line (No attenuation → no reduction in amplitude)

* The actual Voltage at any point on the transmission line is the sum of the incident and reflected wave Voltages at that point. This Voltage wave appears to Standstill on the line

Oscillating in magnitude with time but having fixed positions of maxima and minima Such a wave is known as standing wave



The Voltage equation is

$$E = \frac{E_R(Z_R+R_0)}{2Z_R}\left(e^{j\beta s} + k e^{-j\beta s}\right)$$

$$= \frac{E_R(Z_R+R_0)}{2Z_R}e^{j\beta s} + \frac{E_R(Z_R+R_0)k e^{-j\beta s}}{2Z_R}.$$

Substitute the Value of k

$$E = \frac{E_R(Z_R+R_0)}{2Z_R}e^{j\beta s} + \frac{E_R(Z_R+R_0)}{2Z_R}\left(\frac{Z_R-R_0}{Z_R+Z_0}\right)e^{-j\beta s}$$

$$E = \frac{E_R(Z_R+R_0)e^{j\beta s}}{2Z_R} + \frac{E_R(Z_R-R_0)\,e^{-j\beta s}}{2Z_R}$$

$$= \frac{E_R Z_R e^{j\beta s}}{2Z_R} + \frac{E_R R_0\, e^{j\beta s}}{2Z_R} + \frac{E_R Z_R e^{-j\beta s}}{2Z_R} - \frac{E_R Z_0\, e^{-j\beta s}}{2Z_R}$$

$$= E_R\left[\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right] + I_R R_0\left[\frac{e^{j\beta s} - e^{-j\beta s}}{2}\right]$$

$$= E_R\left[\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right] + jI_R R_0\left[\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right]$$

$$\boxed{E = E_R \cos\beta s + j I_R R_o \sin\beta s} \quad \text{——— (a)}$$

Similarly for current

$$\boxed{I = I_R \cos\beta s + j \frac{E_R}{R_o} \sin\beta s} \quad \text{——— (b)}$$

W.K.T Velocity of propagation $V = \frac{\omega}{\beta}$

$$\beta = \frac{2\pi f}{v}$$

$$\lambda = \frac{v}{f}$$

$$\therefore \beta = \frac{2\pi}{\lambda}$$

(a) $\Rightarrow E = E_R \cos\frac{2\pi s}{\lambda} + j I_R R_o \sin\frac{2\pi}{\lambda} s$

(b) $\Rightarrow I = I_R \cos\frac{2\pi}{\lambda} s + j \frac{E_R}{R_o} \sin\frac{2\pi}{\lambda} s$

CASE (i)

If the line is open circuited, $I_R = 0$

(a) $\Rightarrow E_{oc} = E_R \cos\frac{2\pi}{\lambda} s$

(b) $\Rightarrow I_{oc} = j \frac{E_R}{R_o} \sin\frac{2\pi}{\lambda} s$

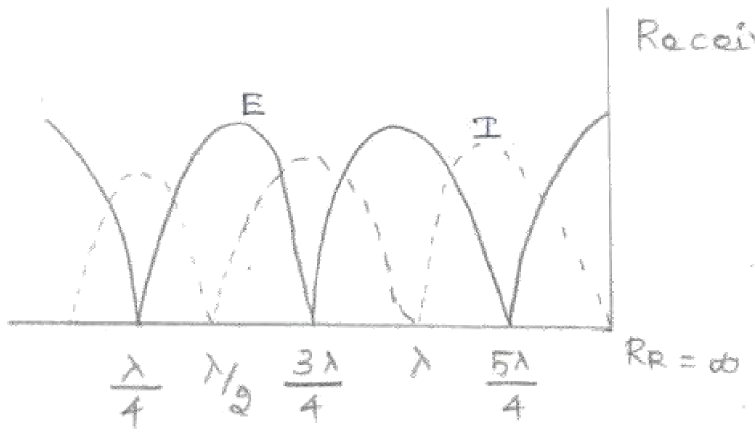


Fig: Voltage and current on a open circuited dissipation loss line

$\therefore$ The Voltage and current are in quadrature everywhere. Thus no power is transmitted along the line

CASE (ii)

If the line is short circuited $E_R = 0$

ⓐ $\Rightarrow E_{sc} = jI_R \sin \frac{2\pi s}{\lambda}$

ⓑ $\Rightarrow I_{sc} = I_R \cos \frac{2\pi}{\lambda} s$

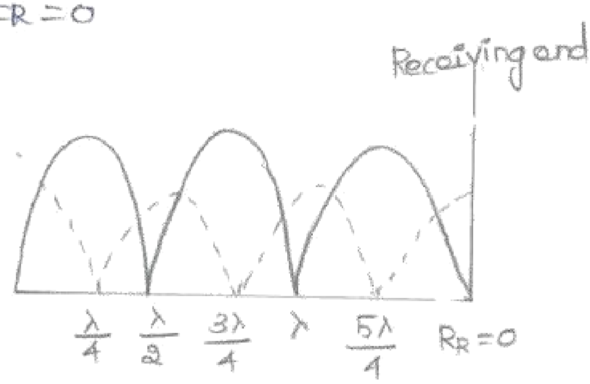


Fig: Voltage and current on a short circuited dissipation less line

* Again the current and Voltage are in quadrature, but the current and Voltage waves have shifted $\frac{\lambda}{4}$ from the position for the open circuit case

Case (iii)

If the line is terminated with $Z_R = R_0$

then $E = \frac{E_R (Z_R + R_0)}{2Z_R} \left[e^{j\beta s} + k e^{-j\beta s} \right]$

$$= \frac{E_R (Z_R + R_0)}{2Z_R} \left[e^{j\beta s} + \left(\frac{Z_R - R_0}{Z_R + R_0} \right) e^{-j\beta s} \right]$$

$$= \frac{E_R (R_0 + R_0)}{2R_0} \left[e^{j\beta s} + \left(\frac{R_0 - R_0}{R_0 + R_0} \right) e^{-j\beta s} \right]$$

$$E = E_R e^{j\beta s}$$

The reflected wave and reflection coefficient becomes zero and a constant Voltage magnitude exists (No attenuation) Continuously Varying Phase angle along the line

Similarly for the current

$$I_R = \frac{I_R(Z_R+R_o)}{2R_o}\left[e^{j\beta s} - k e^{-j\beta s}\right]$$

$$= \frac{I_R(R_o+R_o)}{2R_o}\left[e^{j\beta s} - \left(\frac{R_o-R_o}{R_o+R_o}\right)e^{-j\beta s}\right]$$

$$I_R = I_R e^{j\beta s}$$

Here a constant current magnitude exists

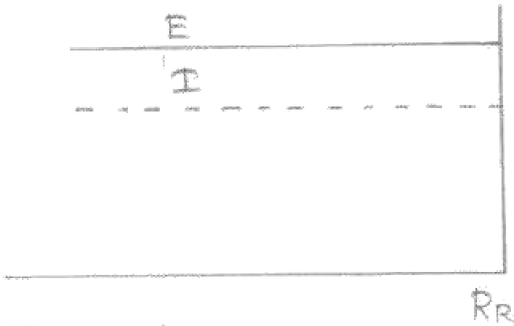


Fig: Voltage and current when $R_R = R_o$

Case(iv)

If the line is terminated in a resistance R_R greater than R_o the reflection coefficient k is positive and the voltage and current conditions on the line will be intermediate to the open-circuit and R_o-terminated condition

For example, If $R_R = 3R_o$

$$k = \frac{R_R - R_o}{3R_R + R_o} = \frac{2R_o}{4R_o}$$

$$E = \frac{E_R(3R_o+R_o)}{6R_o}\left[e^{j\beta s} + \frac{1}{2}e^{-j\beta s}\right]$$

$$= \frac{2E_R}{3}\left[e^{j\beta s} + \frac{1}{2}e^{-j\beta s}\right]$$

Incident Wave $E_{in} = \frac{2}{3} E_R e^{j\beta s}$

Reflected wave $E_{ref} = \frac{E_R}{3} e^{-j\beta s}$

$\therefore$ The Incident wave has an amplitude twice that of reflected wave

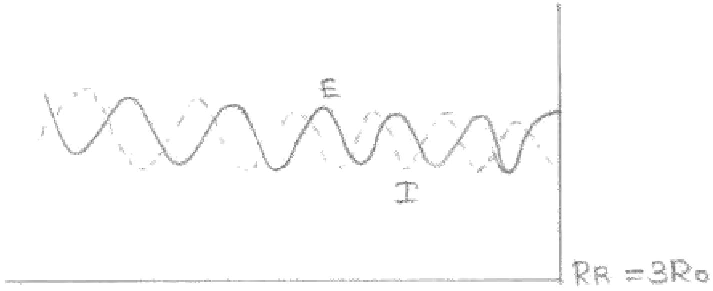


Fig: Voltage & Current on a dissipation less line if load $R_R = 3R_0$

CASE(V)

If the line is terminated in a resistance R_R less than R_0. Assume $R_R = \frac{R_0}{3}$, the Value of k is $\frac{-1}{2}$ and the Phase of the reflected wave is reversed

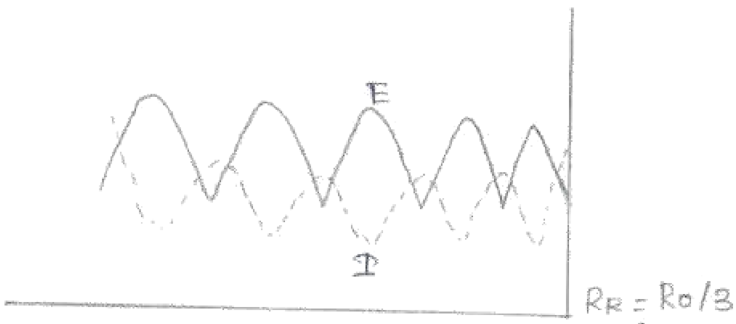


Fig: Voltage and current on a dissipationless line when $R_R = \frac{R_0}{3}$

In general, for resistive loads greater than R_0, the Current and Voltage distribution resembles to that of open-circuited line for resistive loads less than R_0, it resembles to that of Short Circuited line.

2 (ii) DERIVE THE LINE CONSTANTS FOR ZERO DISSIPATION LINE [DISSIPATIONLESS LINE] (6)

At high frequencies, it is assumed that the losses are negligible ie, Line of zero dissipation, The assumption made for a perfect Line is that ω is Large making ωL Large

For a short line, resistance is very small compared with the reactance and G assumed to be zero because of small number of insulators

ADVANTAGES OF ASSUMING DISSIPATION OF ZERO

(i) Analysis is very easier

(ii) Physical interpretation of line performance is possible

The line parameters for the line of zero dissipation are

$$Z = j\omega L \quad (\text{since } R=0)$$

$$Y = j\omega C \quad (G=0 \text{ for dissipationless line})$$

∴ characteristic impedance, $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L}{j\omega C}}$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} \text{ ohms}}$$

Z_0 is wholly resistive and given the symbol R_0

$$\therefore Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

For open-wire line,

The inductance and capacitance of the open wire line at high frequency is given by

$$L = 9.21 \times 10^{-7} \log \frac{d}{a} \text{ henrys/m}$$

$$C = \frac{12.07}{\log \frac{d}{a}} \ \mu\mu f/m$$

The Value of characteristic impedance is

$$R_0 = \sqrt{\frac{9.21 \times 10^{-7} \log \frac{d}{a} \log \frac{d}{a}}{12.07 \times 10^{-12}}}$$

$$R_0 = 276 \log \frac{d}{a} \text{ (ohms)}$$

$$R_0 = \frac{276}{2.303} \ln \frac{d}{a} \text{ (ohms)}$$

$$R_0 = 120 \ln \frac{d}{a} \text{ (ohms)}$$

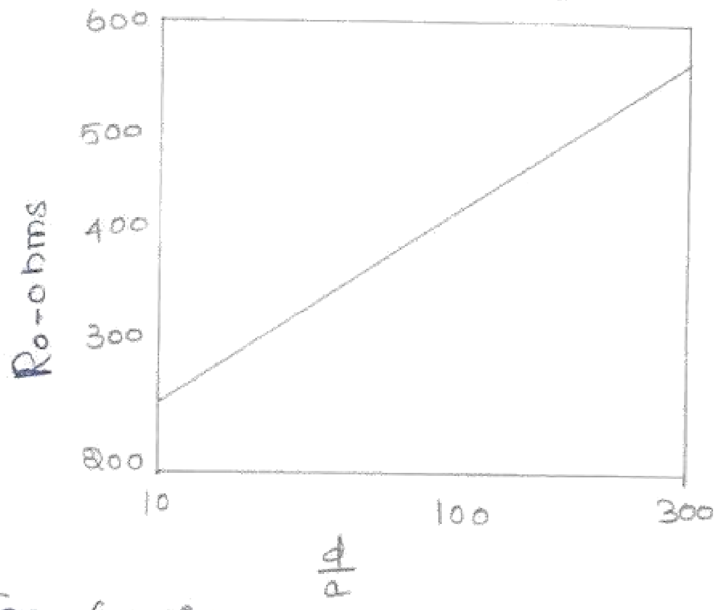


Fig: Variation of R_0 with $\frac{d}{a}$ ratio for an open wire line

For Coaxial line,

The inductance and capacitance of the coaxial line at high frequencies are

$$L = 4.606 \times 10^{-7} \log \frac{b}{a} \text{ henrys/m}$$

$$C = \frac{24.14 \varepsilon_r}{\log \frac{b}{a}} \ \mu\mu f/m$$

The Value of characteristic impedance is

$$R_o = \sqrt{\frac{4.606 \times 10^{-7} \log \frac{b}{a} \log \frac{b}{a}}{24.14\, \varepsilon_r \times 10^{-12}}}$$

$$R_o = \frac{138}{\sqrt{\varepsilon_r}} \log \frac{b}{a} \text{ ohms}$$

$$R_o = \frac{138}{2.303\sqrt{\varepsilon_r}} \ln \frac{b}{a}$$

$$R_o = \frac{60}{\sqrt{\varepsilon_r}} \ln \frac{b}{a} \text{ ohms.}$$

The Value of ε_r is 1 for air Spaced lines

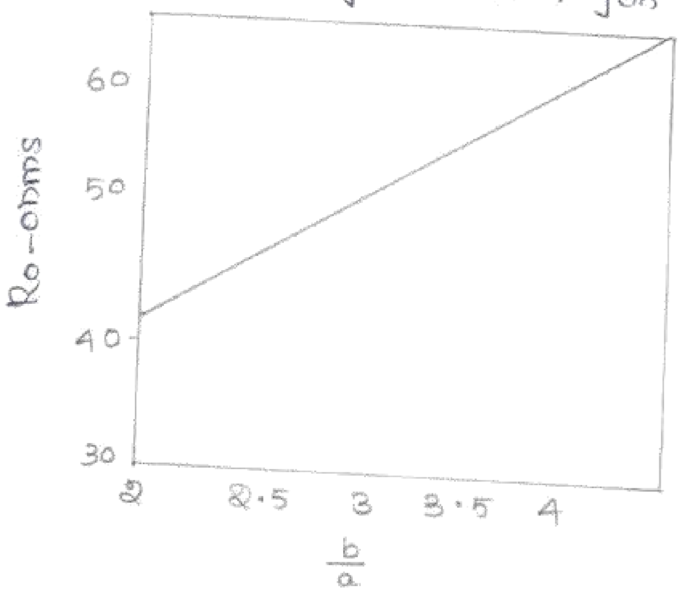


Fig: Variation of Ro with $\frac{b}{a}$ ratio for a Coaxial line.

The propagation Constant γ is

$$\gamma = \sqrt{ZY} = \sqrt{j\omega L \cdot j\omega C}$$

$$= \sqrt{j^2 \omega^2 LC}$$

$$\gamma = j\omega\sqrt{LC}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\therefore \alpha = 0$$

$$\beta = \omega\sqrt{LC} \text{ radians/m}$$

The Velocity of propagation is

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{LC}} \text{ m/sec}$$

Substitude the Value of L and C in Vp for open wire line

$$V_p = \frac{1}{\sqrt{9.21\times10^{-7}\log\frac{d}{a}\times\frac{12.07}{\log\frac{d}{a}}\times10^{-12}}}$$

$$= 3\times10^{8} \text{ m/sec}$$

Thus the Velocity of propagation for the airspaced open-wire dissipationloss line is the Same as the Velocity of linght in Space

For the coaxial line,

$$V_p = \frac{1}{\sqrt{4.606\times10^{-7}\log\frac{b}{a}\times\frac{24.14\,\varepsilon_r\times10^{-12}}{\log\frac{b}{a}}}}$$

$$V_p = \frac{3\times10^{8}}{\sqrt{\varepsilon_r}} \text{ m/sec}$$

Thus the Velocity of propagation in a coaxial line is reduced due to the presence of a dielectric other than air between the Conductors

Thus for a dissipationloss line

$$Z_0 = \sqrt{\frac{L}{C}} \text{ ohms}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC} \text{ radians/m}$$

$$V_p = \frac{1}{\sqrt{LC}} \text{ m/sec.}$$

3 WRITE SHORT NOTES ON THE FOLLOWING (60)

(i) STANDING WAVES (5)

(ii) STANDING WAVE RATIO (5) [Nov/Dec 03, 04, 07, 10]

(iii) MEASUREMENT OF SWR (6) [May/June 05, 06, 09, 12]

(i) STANDING WAVES

The actual Voltage at any point on the transmission line is the Sum of the incident and reflected wave Voltages at the points. This Voltage wave appears to Standstill on the line. Oscillating in magnitude with time but having fixed positions of maxima and minima Such a wave is known as Standing Wave

Standing wave on a dissipationless terminals in a load not equal to Ro is drawn

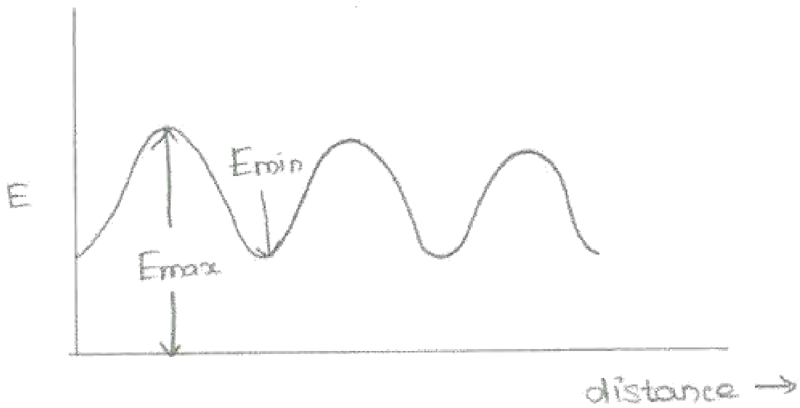


Fig: Standing waves on dissipation loss $Z_R = R_0$

* Standing waves on a line having open or Short Circuit termination is also drawn

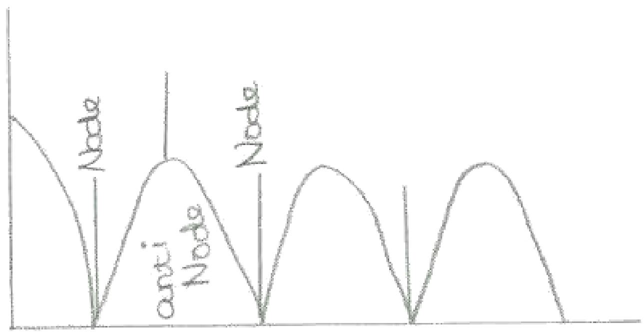


Fig: Standing wave on a dissipation less line having open and Short Circuit termination

Nodes are points of zero Voltage or current in the standing wave systems

Antinodes are points of maximum Voltage or current in the standing wave system. Antinodes also called as loops

A line terminated in R_0 has no standing wave and thus no nodes or loop and is called Smooth line

(ii) STANDING WAVE RATIO (SWR)

The ratio of maximum to minimum magnitudes of current or Voltage on a line having standing waves is called the standing wave ratio (s)

$$S = \left|\frac{E_{max}}{E_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

RELATION BETWEEN SWR AND REFLECTION COEFFICIENT

From the Voltage equation,

$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[e^{j\beta s} + K e^{-j\beta s}\right]$$

The maxima of Voltage occur at a points at which the incident and reflected wave are inphase

$$E_{max} = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[1+|K|\right]$$

Similarly the manima Voltage occur at a points at which the incident and reflected wave are out of phase (opposite sign)

$$E_{min} = \frac{E_R(Z_R+Z_0)}{2Z_R}\left[1-|K|\right]$$

Then the standing wave ratio is

$$S = \left|\frac{E_{max}}{E_{min}}\right| = \frac{1+|K|}{1-|K|}$$

$$\boxed{S = \frac{1+|K|}{1-|K|}} \quad \text{——— (1)}$$

<u>To</u> <u>FIND</u> <u>k</u>

$$S(1-|K|) = 1+|K|$$

$$S - S|K| = 1+|K|$$

$$|K| + S|K| = S-1$$

$$|K|\left[1+S\right] = S-1$$

$$\boxed{|K| = \frac{S-1}{S+1}} \quad \text{——— (2).}$$

$$S = \left|\frac{E_{max}}{E_{min}}\right|$$

$$\therefore |K| = \frac{\left|\frac{E_{max}}{E_{min}}\right|-1}{\left|\frac{E_{max}}{E_{min}}\right|+1}$$

$$= \frac{|E_{max}|-|E_{min}|}{|E_{max}|+|E_{min}|}$$

From (1) and (2) it is possible to calculate values of $|K|$ and S from the measurements of maximum and minimum voltages on the line.

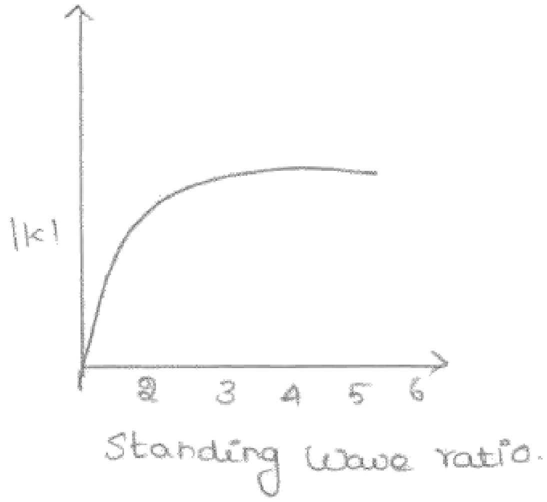


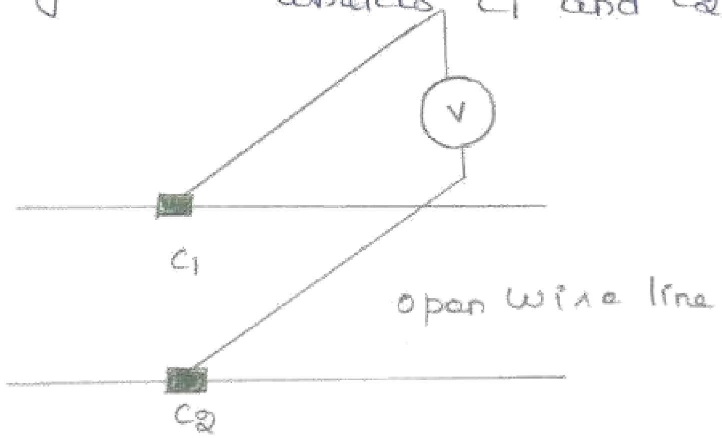
Fig: Relation between the Standing wave ratios and |k|

(iii) MEASUREMENT OF SWR

SWR can be calculated if Emax and Emin are given Thus, determination of SWR is the determination of Emax and Emin.

(a) OPEN WIRE LINES

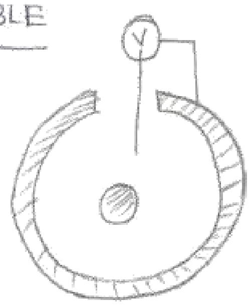
* The Value of Emax and Emin can be obtained on open-wire lines by arranging a Simple Setup
* A Sliding Contact Voltmeter (V) is used to measure the Voltage at different points along the line by touching the Contacts C_1 and C_2



* Generally high impedance Vaccum tube Voltmeter is used ~~to~~ for the measurement of Voltages.
* whenever the Voltmeter reads maximum Voltage along the line, its reading will ~~be~~ give the Value of Vmax.

Several Values of Vmax are taken and the average of all Vmax gives the find Value of Vmax. Similarly, whenever the Voltmeter reads minimum its reading will give the Value of Vmin. This we can calculate SWR

(b) COAXIAL CABLE



For Coaxial lines, we have to use a Certain length of Coaxial line in which a longitudinal Slot, a half wavelength or more long, has been cut. A wire probe is inserted into the air dielectric of the line as a pickup device. A Vaccum tube Voltmeter or other detector is Connected between probe and Sheath (Convering) as an Indicator. If the meter provides linear Indications, S is readily determined. If the meter is non-linear, Corrections must be applied to readings obtained.

For a Special case of resistive load ie $Z_R = R_R$

$$S = \frac{1+|k|}{1-|k|} = \frac{1+\left(\frac{R_R - R_0}{R_R + R_0}\right)}{1-\left(\frac{R_R - R_0}{R_R + R_0}\right)}$$

$$= \frac{\frac{R_R + R_o + R_R - R_o}{R_R + R_o}}{\frac{R_R + R_o - R_R + R_o}{R_R + R_o}}$$

$$= \frac{2R_R}{2R_o}$$

$$= \frac{R_R}{R_o}$$

$$S = \frac{R_R}{R_o} \text{ if } R_R >> R_o$$

$$S = \frac{R_o}{R_R} \text{ if } R_o >> R_R$$

4 EXPLAIN IN DETAIL ABOUT

(i) POWER MEASUREMENT

(ii) IMPEDANCE MEASUREMENT

On a line ? [May /June 2006] [Nov/Dec 2003, 2004, 2005, 11]

(i) MEASUREMENT OF POWER

The Voltage and current on the dissipation loss line is given by

$$E = \frac{I_R (Z_R + Z_o)}{2} (1 + |k| \angle \phi - 2\beta s)$$

$$I = \frac{I_R (Z_R + Z_o)}{2R_o} (1 - |k| \angle \phi - 2\beta s)$$

For a Voltage maximum, the incident and the reflected waves are inphase. $I\angle 0°$ is proportional to the incident wave Voltage $|k|\angle\phi - 2\beta s$ is proportional to the reflected Voltage

$$E_{max} = \text{Inphase Condition} = \frac{I_R(Z_R+Z_0)}{2}(1+|k|)$$

Similar resoning show that at a Current maximum the incident and reflected wave must be inphase, So that

$$I_{max} = \frac{I_R|Z_R+Z_0|}{2R_0}(1+|k|)$$

$$\therefore \frac{E_{max}}{I_{max}} = R_0.$$

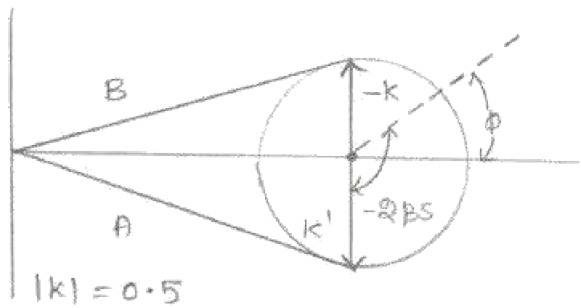


Fig: Diagram Illustrating Equation

Since a change to the Value at Voltage and Current minima requires only the reverse of phase of the reflected waves or minus Sign in front of $|k|$, the ratio of $\frac{E_{min}}{I_{min}}$ is given by

$$\frac{E_{min}}{I_{min}} = R_0$$

It can be Seen that a Voltage maximum and a current minimum occur at the Same point on the line as seen in standing wave waveform. Thus the impedance looking into the line towards load is purely resistive and is given by

$$I_{min} = \frac{I_R |Z_R + Z_0|}{2R_0}(1-|K|)$$

The resistive impedance seen at a Voltage loop (Antinode) is

$$\frac{E_{max}}{I_{min}} = R_0\left[\frac{1+|K|}{1-|K|}\right] = SR_0 = R_{max}$$

Since the Voltage and current are again in phase at a current loop, the resistive impedance may be identified as R_{min}

$$\frac{E_{min}}{I_{max}} = \frac{R_0(1-|K|)}{(1+|K|)} = \frac{R_0}{S} = R_{min}$$

The power passing a Voltage loop is the power effectively flowing into a resistance R_{max} at Voltage E_{max} so that

$$P = \frac{E^2_{max}}{R_{max}}$$

The same value of power must also pass the current loop, effectively flowing into a resistance R_{min} at Voltage E_{min}, since there is no line dissipation so that

$$P = \frac{E^2_{min}}{R_{min}}$$

Multiplying the above two equations for power

$$P^2 = \frac{E^2_{max} \cdot E^2_{min}}{R_{max} \cdot R_{min}}$$

Substituting the value of E_{max}, E_{min}, R_{max}, R_{min} the power flow along the line is given by

$$P^2 = \frac{E^2_{max} \cdot E^2_{min}}{\left[\frac{E_{max}}{I_{min}}\right] \cdot \left[\frac{E_{min}}{I_{max}}\right]} = \frac{E^2_{max} \cdot E^2_{min}}{SR_0 \cdot \frac{R_0}{S}}$$

$$P^2 = \frac{E^2_{max} \cdot E^2_{min}}{R_0^2}$$

$$\boxed{P = \frac{|E_{max}| \cdot |E_{min}|}{R_0}}$$

Similarly $\boxed{P = |I_{max}|\,|I_{min}|\,R_0}$

* Last two equations permit easy measurement of power flow on a line of negligible losses

(ii) IMPEDANCE MEASUREMENT

The unknown value of a load impedance Z_R connected to a transmission line may be determined by standing wave measurement on the open wire (or) slotted line. Bridge circuit is used for the measurement of unknown impedance

At the point of voltage minimum at a distance s' from the load it can be shown that

$$Z_S = R_{min} = \frac{R_0}{S} \qquad S \rightarrow SWR$$

At any point on the line, the input impedance is given by

$$Z_s = R_0' \left[\frac{Z_R + jR_0 \tan\left(\frac{2\pi s'}{\lambda}\right)}{R_0 + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right)}\right] = \frac{R_0}{S}$$

Solving for Z_R gives

$$R_0 + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right) = S\left[Z_R + jR_0 \tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$$-SZ_R + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right) = -R_0 + jR_0\, S\tan\left(\frac{2\pi s'}{\lambda}\right)$$

$$-Z_R\left[S - j\tan\left(\frac{2\pi s'}{\lambda}\right)\right] = -R_0\left[1 - jS\tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$$Z_R = R_0\left[\frac{1 - jS\tan\left(\frac{2\pi s'}{\lambda}\right)}{S - j\tan\left(\frac{2\pi s'}{\lambda}\right)}\right]$$

gives the value of connected load impedance

$$Z_R = R_0\left[\frac{1 - jS\tan\beta s'}{S - j\tan\beta s'}\right]$$

where $\beta = \frac{2\pi}{\lambda}$

5) DERIVE INPUT IMPEDACE OF DISSIPATIONLESS LINE AND ALSO DERIVE THE OPEN AND SHORT CIRCUITED LINE IMPEDACE (OR)

* DISCUSS THE THEORY OF OPEN & SHORT CIRCUITED LINE WITH VOLTAGE AND CURRENT DISTRIBUTION DIAGRAM (OR) [Dec-2004, 2005, 2006, 2011, 2012, 13]

* OBTAIN THE EXPRESSION FOR INPUT IMPEDANCE OF OPEN AND ~~CI~~ SHORT CIRCUITED LINE [May-18]

Input Impedance of Open and Short Circuited Lines

The input impedance of the dissipationless line is

$$Z_s = \frac{E_s}{I_s}$$

$$= \frac{E_R \cos\beta s + j I_R R_o \sin\beta s}{I_R \cos\beta s + j \frac{E_R}{R_o} \sin\beta s}$$

$\div$ by $\cos\beta s$

$$Z_s = \frac{E_R + j I_R R_o \tan\beta s}{I_R + j \frac{E_R}{R_o} \tan\beta s}$$

$$E_R = Z_R I_R$$

$$\therefore Z_s = \frac{Z_R I_R + j I_R R_o \tan\beta s}{\frac{R_o I_R + j I_R Z_R \tan\beta s}{R_o}}$$

$$\boxed{Z_s = R_o \left[\frac{Z_R + j R_o \tan\beta s}{R_o + j Z_R \tan\beta s}\right]}$$

For a short circuited line $Z_R = 0$

$$Z_s = Z_{sc} = j R_o \tan\beta s$$

$$\beta = \frac{2\pi}{\lambda}$$

$$Z_{sc} = j R_o \tan \frac{2\pi s}{\lambda}$$

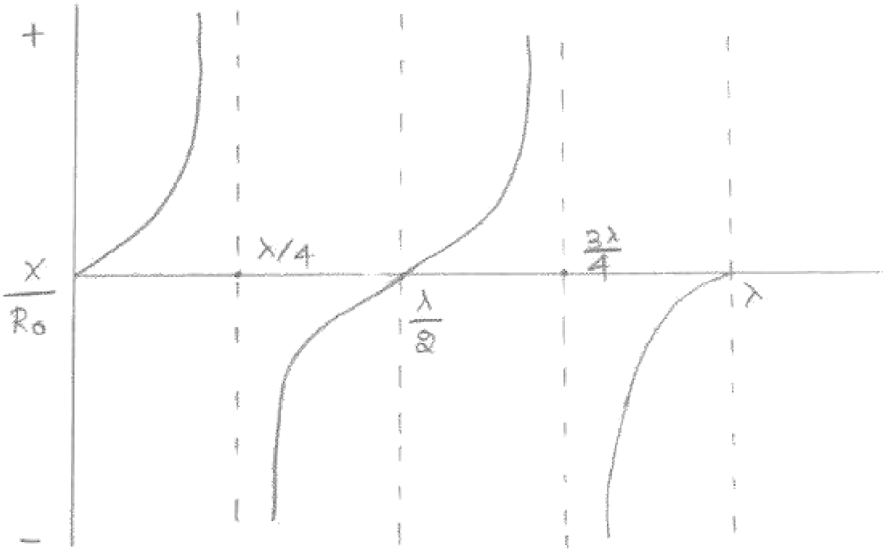


Fig : Short circuited line

The above figure represents the Variation of input impedance of dissipationless line as a function of length for a Short Circuited line

$$Z_s = R_o \left[\frac{1 + j\frac{R_o}{Z_R} \tan\beta s}{\frac{R_o}{Z_R} + j \tan\beta s} \right]$$

For an open circuited line $Z_R = \infty$

$$Z_{oc} = R_o \times \frac{1}{j \tan\beta s}$$

$$= -jR_o \cot\beta s$$

$$\beta = \frac{2\pi}{\lambda}$$

$$Z_{oc} = -jR_o \cot \frac{2\pi s}{\lambda}$$

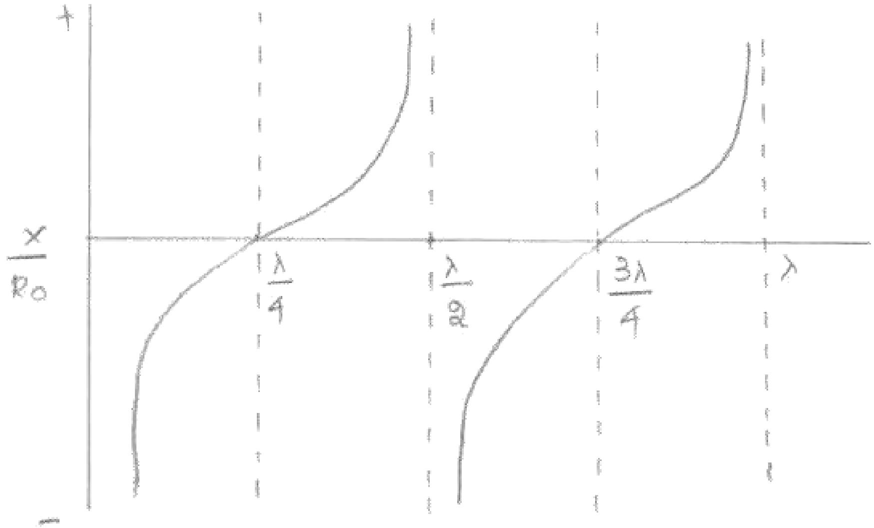


Fig: open circuited line

The above figure represents the Variation of input impedance of dissipationless line as a function of length for open circuited line

From the equation of input impedance of either an open or short circuited line, it can be seen that the input impedance is a pure reactance. That reactance may be positive or negative

In the above 2 figures, for the first quater wave length a short circuited line act as an inductance where as an open circuited line appears as a capacitance. These reactances reverse each quator wave length

PROBLEM

1. A Co-axial cable is made up of copper with conductivity 5.76×10^{7} ℧/m. The diameter of inner conductor and outer conductor are 4 mm and 16mm respectively. The thickness of the outer conductor is 1mm. The space between conductor is filled with a dielectric of relative permittivity of 4.

Calculate 1. Inductance 2. capacitance C

3. DC resistance Rdc 4. AC resistance Rac

Assume frequency of transmitted signal 150KHz

Soln

d_1 = Diameter of inner conductor = 4mm, a = 2mm = Radius

d_2 = Diameter of outer conductor = 16mm, b = 8mm = Radius

t = Thickness of outer conductor = 1m

C = Outer radius of outer conductor = b+t = 9×10^{-3} m

$\varepsilon_r = 4$, $f = 150 \times 10^{3}$ Hz

1. For co-axial cable, the inductance is given by,

$$L = 4.61 \times 10^{-7} \log_{10}\left(\frac{b}{a}\right)$$

$$L = 4.61 \times 10^{-7} \log_{10}\left(\frac{8 \times 10^{-3}}{2 \times 10^{-3}}\right)$$

$$\boxed{L = 0.27755\,\mu H/m}$$

2. For co-axial cable, the capacitance is given by

$$C = \frac{24.13 \times 10^{-12}(\varepsilon_r)}{\log_{10}\left(\frac{b}{a}\right)} \text{ F/m}$$

$$C = \frac{24.13 \times 10^{-12} \times 4}{\log_{10}\left(\frac{8 \times 10^{-2}}{2 \times 10^{-12}}\right)}$$

$$\boxed{C = 0.1603\,nF/m}$$

3) The d.c resistance is given by

$$R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2} + \frac{1}{c^2 - b^2}\right]$$

$$= \frac{1}{\pi \times 5.76\times10^7}\left[\frac{2}{(2\times10^{-3})^2} + \frac{1}{(9\times10^{-3})^2 - (8\times10^{-3})^2}\right]$$

$$\boxed{R_{dc} = 1.7066\times10^{-3}\ \Omega/m}$$

4) The a.c resistance is given by,

$$R_{ac} = 4.17\times10^{-8}\sqrt{f}\left[\frac{1}{a} + \frac{1}{b}\right]$$

$$= 4.17\times10^{-8} \times \sqrt{150\times10^3}\left[\frac{1}{2\times10^{-3}} + \frac{1}{8\times10^{-3}}\right]$$

$$\boxed{R_{ac} = 10.0939\times10^{-3}\ \Omega/m}$$

2. A line with zero dissipation has $R = 0.006\ \Omega/m$, $L = 2.5\ \mu H/m$ and $C = 4.45\ PF/m$. If the line is operated at 10MHz find i) R_0 ii) α iii) β iv) v v) λ

Soln

Given $R = 0.006\ \Omega/m$, $L = 2.5\times10^{-6}\ H/m$,

$C = 4.45\ pF/m$, $F = 10MHz$. At $f = 10MHz$,

$$\omega L = 2\pi f L = 2\times\pi\times10\times10^6\times2.5\times10^{-6}$$

$$= 15.708\ \Omega.$$

Hence $\omega L >> R$ at 10MHz

So according to standard assumption for the dissipationless line, we can neglect R.

(i) The characteristic impedance is given by

$$Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{2.5 \times 10^{-6}}{4.45 \times 10^{-12}}}$$

$$\boxed{Z_0 = 749.53\,\Omega}$$

(ii) The propagation constant is given by

$$\gamma = \alpha + j\beta$$

$$\gamma = 0 + j\omega\sqrt{LC}$$

Hence $\gamma = 0 + j(2 \times \pi \times 10 \times 10^6)\sqrt{2.5 \times 10^{-6} \times 4.45 \times 10^{-12}}$

$$\gamma = \cancel{\alpha + j\beta}$$

$$\boxed{\gamma = 0 + j0.2095 \text{ per m}}$$

∴ Attenuation Constant $= \alpha = 0$ and

phase Constant $= \beta = 0.2095$ rad/m

iii) The Velocity of propagation is given by

$$v = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2.5 \times 10^{-6} \times 4.45 \times 10^{-12}}}$$

$$\boxed{v = 2.998 \times 10^8 \text{ m/sec}}$$

iv) The wave length is given by

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2095}$$

$$\boxed{\lambda = 29.9913 \text{ m}}$$

3. A low loss transmission line of 100 Ω characteristic impedance is connected to a load of 200 Ω. Calculate the Voltage reflection coefficient and the Standing wave ratio. (6 mark) [May - 12]

Soln

Given

$Z_0 = R_0 = 100\,\Omega$

$Z_R = 200\,\Omega$

The reflection coefficient is given by,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{200 - 100}{200 + 100}$$

$$= \frac{100}{300}$$

$$\boxed{K = 0.3333}$$

The Standing wave ratio is given by,

$$S = \frac{1 + |K|}{1 - |K|}$$

$$= \frac{1 + 0.3333}{1 - 0.3333}$$

$$\boxed{\begin{array}{l} S = 1.99985 \\ S \approx 2 \end{array}}$$

4. What are the Special Considerations of radio frequency line? A radio frequency line with $Z_0 = 70\,\Omega$ is terminated by $Z_L = 115 - j80\,\Omega$ at $\lambda = 2.5m$. Find the VSWR and maximum and minimum line impedances.

[Nov/Dec 2007] [Mark 4]

Soln

Given $Z_0 = 70\,\Omega$ $\quad Z_R = 115 - j80$

The reflection Coefficient k is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{(115 - j80) - 70}{(115 - j80) + 70}$$

$$= \frac{45 - j80}{185 - j80}$$

$$= \frac{91.7877 \angle -60.64^\circ}{201.5564 \angle -23.38^\circ}$$

$$\boxed{K = 0.4553 \angle -37.26^\circ}$$

Hence VSWR is given by

$$S = \frac{1+|k|}{1-|k|} = \frac{1 + 0.4553}{1 - 0.4553}$$

$$\boxed{S = 2.6717}$$

Maximum line impedance is given by

$$(Z_{(max)}) = S Z_0 = S R_0$$

$$= (2.6717)(70)$$

$$\boxed{Z_{s(max)} = 187.08\,\Omega}$$

Minimum line impedance is given by

$$Z_{s(min)} = \frac{Z_0}{S} = \frac{R_0}{S}$$

$$= \frac{70}{(2.6717)} = 26.2\,\Omega$$

$$\boxed{Z_{s(min)} = 26.2\,\Omega}$$

5. A lossless line has a standing wave ratio of 4. The R_0 is 150 ohm and the maximum, Voltage measured on the line is 135V. Find the power being delivered to the load. Derive the equation used

Soln

At Voltage maxima the impedance is maximum and is given by

$$R_{max} = SR_0 = 4(150) = 600\,\Omega$$

Thus the power delivered to the load is given by

$$P = \frac{E^2_{max}}{R_{max}}$$

$$= \frac{(135)^2}{600}$$

$$\boxed{P = 30.375W}$$