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EC3452-ELECTROMAGNETIC FIELDS
(Regulation 2021)

Unit-V Uniform Electromagnetic Waves

Part A - Two Marks

1. Define a wave.

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location then the group of phenomena constitutes a wave.

2. Mention the properties of uniform plane wave.

i) At every point in space, the electric field E and magnetic field H are perpendicular to each other. ii) The fields vary harmonically with time and at the same frequency everywhere in space.

3. Define intrinsic impedance or characteristic impedance.

It is the ratio of electric field to magnetic field or It is the ratio of square root of permeability to permittivity of medium.

4. Give the characteristic impedance of free space.

377ohms.

5. Define propagation constant.

Propagation constant is a complex number $\gamma = \alpha + j\beta$ where γ is propagation constant.

6. Define skin depth

It is defined as that depth in which the wave has been attenuated to 1/e or approximately 37% of its original value.

7. Define Poynting vector. T

he pointing vector is defined as rate of flow of energy of a wave as it propagates. P =E X H

8. State Poynting's Theorem.

The net power flowing out of a given volume is equal to the time rate of decrease of the energy stored within the volume conduction losses.

9.Give the difficulties in FDM.

FDM is difficult to apply for problems involving irregular boundaries and non-homogeneous material properties.

10. Explain the steps in finite element method.

i) Discrimination of the solution region into elements. ii) Generation of equations for fields at each element. iii) Assembly of all elements. iv) Solution of the resulting system.

11. State Maxwell's fourth equation.

The net magnetic flux emerging through any closed surface is zero.

12. State Maxwell's Third equation

The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

13. State the principle of superposition of fields.

The total electric field at a point is the algebraic sum of the individual electric field at that point.

14. Define loss tangent.

Loss tangent is the ratio of the magnitude of conduction current density to displacement current density of the medium.

15. Define reflection coefficient.

Reflection coefficient is defined as the ratio of the magnitude of the reflected field to that of the incident field.

16. Define transmission coefficient

Transmission coefficient is defined as the ratio of the magnitude of the transmitted field to that of incident field.

17. What will happen when the wave is incident obliquely over dielectric dielectric boundary?

When a plane wave is incident obliquely on the surface of a perfect dielectric part of the energy is transmitted and part of it is reflected .But in this case the transmitted wave will be refracted, that is the direction of propagation is altered.

18. What are uniform plane waves?

Electromagnetic waves which consist of electric and magnetic fields that are perpendicular to each other and to the direction of propagation and are uniform in plane perpendicular to the direction of propagation are known as uniform plane waves.

19. What is the significant feature of wave propagation in an imperfect dielectric?

The only significant feature of wave propagation in an imperfect dielectric compared to that in a perfect dielectric is the attenuation undergone by the wave.

20. What is the major drawback of finite difference method?

The major drawback of finite difference method is its inability to handle curved boundaries accurately.

21. Define power density.

The power density is defined as the ratio of power to unit area. Power density=power/unit area.

22. What is Normal Incidence?

When a uniform plane wave incidences normally to the boundary between the media then it is known as normal incidence.

23. What is called attenuation constant?

When a wave propagates in the medium, it gets attenuated. The amplitude of the signal reduces. This is represented by attenuation constant α. It is measured in Neper per meter (NP/m). But practically it is expressed in decibel (dB).

24. What is phase constant?

When a wave propagates, phase change also takes place. Such a phase change is expressed by a phase constant β . It is measured in radian per meter (rad/m).

25. How voltage maxima and minima are separated?

In general voltage minima are separated by one half wavelength. Also the voltage maxima are also separated by one half wave length.

PART-B

1) Explain the Poynting's Theorem.

* At every point in E & H are $\perp^r$ to each other & to the direction of travel

* The fields are vary harmonically with time at the same frequency, everywhere in space

* Each field has same direction, magnitude & phase at every point in any plane $\perp^r$ to the direction of wave travel

Poyntings theorem

- EMW an energy can be transported from transmitter to receiver. The energy stored in an electric & magnetic field is transmitted at a certain rate of energy flow which can be calculated with help of Poynting theorem.

E is electric field expressed in V/m, H is magnetic field expressed in A/m.

- If we take the product of 2 fields it gives a new quantity which is expressed as W/unit area is called power density. As E & H both are vectors, to get power density we carry out either dot or cross product. The result of dot product is always

The power density is given by

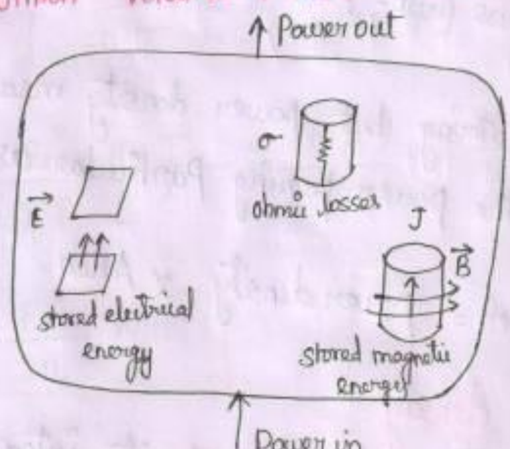
$$\vec{P} = \vec{E} \times \vec{H}$$

where $\vec{P}$ is called poynting Vector.

- It is an instantaneous power density vector associated with EMF at a given point.

- poynting theorem is based On law of conservation of energy in electromagnetism. poynting theorem Can be stated as follows.

"The net power flowing out of given Volume V is equal to the time rate of decreases in the energy Stored within Volume V minus ohmic power dissipated.



Let $\vec{E} = E_x \vec{a_x}$ & $\vec{H} = H_y \vec{a_y}$

$$\vec{P} = \vec{E} \times \vec{H} = E_x \vec{a_x} \times H_y \vec{a_y} = E_x H_y \vec{a_z}$$

$$\vec{P} = P_z \vec{a_z}$$

The above equation indicates E, H & P are mutually ⊥ to each other.

consider that electric field propagates in free space given by

$$\vec{E} = [E_m \cos(\omega t - \beta z)] \vec{a_x}$$

$$\eta = \eta_0 = \frac{E_m}{H_m} = 120\pi = 377\Omega$$

Where $\eta = \eta_0$ is called Intrinsic impedance

$$\vec{H} = [H_m \cos(\omega t - \beta z)] \vec{a_y}$$

$$= \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z)\right] \vec{a_y}$$

$$\vec{P} = \vec{E} \times \vec{H} = E_m \cos(\omega t - \beta z) \vec{a_x} \times \frac{E_m}{\eta_0} \cos(\omega t - \beta z)$$

$$\vec{P} = \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z) \vec{a_z}$$

This is nothing but power density measured in W/m^2. Thus the power passing particular area is given by

Power = power density × Area.

Average power density

To find average power density integrate power density in z direction over one cycle & divide by the period T of one cycle.

2) Problem:

*) In free space $H = 0.2\cos(\omega t - \beta x)\,\vec{a_x}$ A/m. Find the total power passing through a circular disc of radius 5cm

(42)

$$H = 0.2\cos(\omega t - \beta x)\,\vec{a_x}$$

$$r = 5cm = 5\times10^{-2}m = 0.05m$$

$$\eta = 120\pi$$

Area $A = \pi r^2 = \pi(0.05)^2$

$$\eta = \frac{E}{H} \Rightarrow E = \eta H = 120\pi$$

$$E = 75.4 \text{ V/m}$$

Average power $P_{av} = 1/2\, EH$

$$P_{av} = 1/2 \times 75.4 \times 0.2 = 7.54 \text{ W/m}^2$$

Total power $P = P_{av} \times \text{Area}$

$$= 7.54 \times \pi \times (0.05)^2$$

$$= 0.059\text{W or } 59\text{mW}$$

3) Explain plane waves in lossless medium.

Plane waves in various media:-

A media in electromagnetics is characterized by three parameters ε, μ and σ

1. Lossless medium:-

In a lossless medium, ε and μ are real $\sigma=0$, so β is real $\therefore \gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$$\gamma^2 = j^2 \omega^2 \mu\varepsilon = (j\beta)^2 \Rightarrow \beta = \omega\sqrt{\mu\varepsilon}$$

Assume the electric field with

*) only x-component,

*) No variation along x-and y-axis and

*) propagation along z-axis.

ie $\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = 0$

Helmholtz wave equation reduces to

$$\frac{\partial^2}{\partial z^2} E_x + \beta^2 E_x = 0$$

whose solution gives wave in one dime[nsion] as follows.

$$E_x = E^+ e^{-j\beta z} + E^- e^{+j\beta z}$$

where, E^+ and E^- are arbitrary const[ants]

Putting in the time dependence and taking real part, we get,

$$E_x(z,t) = E^+ \cos(\omega t - \beta z) + E^- \cos(\omega t + \beta z)$$

For constant phase,

$$\omega t - \beta z = \text{constant} = b \text{ (say)}$$

Since phase velocity,

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{\omega t - b}{\beta}\right) = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$= \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$$

$$\boxed{\therefore \beta = \omega\sqrt{\mu\varepsilon}}$$

For free space,

$$v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \times 10^8 \text{ m/s}$$

which is the speed of light in free space.

This emergence of speed of light from electromagnetic considerations is one of the main contributions from Maxwell's theory.

The magnetic field can be obtained from the source free Maxwell's curl equation.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\vec{H} = -\frac{\nabla \times \vec{E}}{j\omega\mu} = \frac{j\nabla \times \vec{E}}{\omega\mu}$$

$$= \frac{j}{\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E^{+}e^{-j\beta z} + E^{-}e^{+j\beta z} & 0 & 0 \end{vmatrix}$$

$$= \hat{y}\,\frac{j}{\omega\mu}\left\{\frac{\partial}{\partial z}\left(E^{+}e^{-j\beta z} + E^{-}e^{+j\beta z}\right)\right\}$$

$$\vec{H} = \frac{-j\beta\left(E^{+}e^{-j\beta z}\right) + \left(E^{-}e^{+j\beta z}\right)j\beta}{\omega\mu}(j)\,\hat{y}$$

$$= \frac{-j\beta\left\{\left(E^{+}e^{-j\beta z}\right) - \left(E^{-}e^{+j\beta z}\right)\right\}}{\omega\mu}(j)\,\hat{y}$$

$$= \frac{\beta\left\{\left(E^{+}e^{-j\beta z}\right) - \left(E^{-}e^{+j\beta z}\right)\right\}}{\omega\mu}\,\hat{y}$$

$$\boxed{\vec{H} = \frac{1}{\eta}\left[E^{+}e^{-j\beta z} - E^{-}e^{+j\beta z}\right]\hat{y}}$$

where, η is the wave impedance of the plane wave

$$\eta = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\varepsilon}} = \frac{|E_x|}{|H_y|}$$

For free space,

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$$

4) Explain the plane waves in Lossy medium.

2. Lossy conducting medium:- (7)

If the medium is conductive with a conductivity σ, then the maxwell's curl equation can be written as

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}.$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} + \sigma\vec{E} = (j\omega\varepsilon + \sigma)\vec{E} = j\omega\varepsilon_{eff}\vec{E}$$

$$\varepsilon_{eff}(\omega) = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - \frac{j\sigma}{\omega} = \varepsilon\left(1 - \frac{j\sigma}{\omega\varepsilon}\right)$$

The effect of the conductivity has been absorbed in the complex, frequency dependent effective permittivity

$$\nabla^2\vec{E} + \omega^2\mu\,\varepsilon_{eff}(\omega)\vec{E} = \nabla^2\vec{E} + (j\gamma)^2\vec{E} = 0$$

we can define a complex propagation constant

$$\gamma = j\omega\sqrt{\mu\,\varepsilon_{eff}(\omega)} = \alpha + j\beta$$

where, α is the attenuation constant and β is the phase constant

what is implication of complex wave vector?

✓ The wave is exponentially decaying

✓ The dispersion relation for a conductor (usually non-magnetic) is

$$\gamma = j\omega\sqrt{\mu\varepsilon_{eff}(\omega)} = j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{\frac{\varepsilon_{eff}(\omega)}{\varepsilon_0}}$$

$$= j\omega\sqrt{\mu_0\varepsilon_0}\,n_{eff}(\omega) = j\frac{\omega}{c}n_{eff}(\omega)$$

where, η_{eff} is the complex refractive inde[x]

1-D wave equation for general lossy medi[um] becomes

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

whose solution is 1-D plane waves as follows,

$$E_x(z) = E^+ e^{-\gamma z} + E^- e^{+\gamma z}$$

$$= E^+ e^{-\alpha z} e^{-j\beta z} + E^- e^{\alpha z} e^{j\beta z}$$

putting the time dependence and taking real part, we get,

$$E_x(z,t) = E^+ e^{-\alpha z} \cos(\omega t - \beta z) + E^- e^{\alpha z} \cos(\omega t + \ldots$$

The magnetic field can be found out from maxwells equations as in the previous sect[ion]

$$H_y(z) = \frac{1}{\eta_{eff}} [E^+ e^{-\gamma z} - E^- e^{\gamma z}]$$

where useful expression for intrinsic imped[ance] is

$$\eta_{eff} = \frac{j\omega\mu_0}{\gamma} = \frac{j\omega\mu_0}{j\omega\sqrt{\mu_0 \epsilon_{eff}(\omega)}}$$

$$\boxed{\eta_{eff} = \sqrt{\frac{\mu_0}{\epsilon_{eff}(\omega)}}}$$

For good dielectric,

$\therefore \sigma << \omega\varepsilon \therefore \gamma = j\omega\sqrt{\mu\varepsilon}\left(\sqrt{1-\frac{j\sigma}{\omega\varepsilon}}\right)$

It can be approximated using Taylor's series expansion obtain α and β as follows

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}, \quad \boxed{\beta = \omega\sqrt{\mu\varepsilon}}$$

For a good conductor,

$$\boxed{\sigma >> \omega\varepsilon}$$

$$\therefore \gamma \cong (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\boxed{\Rightarrow \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Skin effect:-

*) The field do attenuate as they travel in a good dielectric medium

*) α in a good dielectrics is very small in comparison to that of a good conductor

*) As the amplitude of the wave varies with $e^{-\alpha z}$

*) The wave amplitude reduces it by $1/e$ or 37% times over a distance of

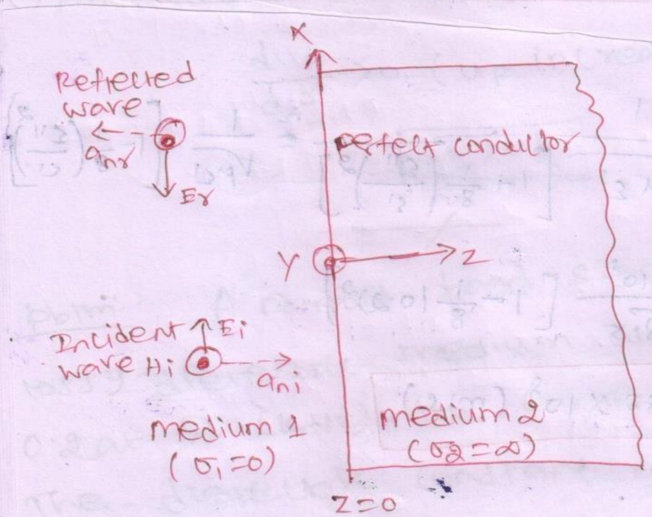
5) Explain the normal incident at a plane conducting medium.

For a high-loss dielectric, ε'' will be a function of ω and may have a magnitude comparable to ε'.

Normal Incidence at a Plane Conducting Boundary

*) The incident wave travels in a lossless medium

*) The boundary is an interface with a perfect conductor.



Incident wave (inside medium 1)

$$\vec{E}_i(z) = \hat{a}_x E_{io} e^{-j\beta_1 z}$$

$$\vec{H}_i(z) = \hat{a}_y \frac{E_{io}}{\eta_1} e^{-j\beta_1 z}$$

Where, E_{io} is the magnitude of $\vec{E}_i$

β_1 is the phase constant

η_1 is the intrinsic impedance of medium 1

Inside wave in medium 2, both electric and magnetic fields vanish. $\vec{E_2} = 0$, $\vec{H_2} = 0$

No wave is transmitted across the boundary into the $z > 0$

Reflected wave (inside medium 1)

$$\vec{E_r}(z) = \hat{a_x} E_{ro} e^{+j\beta_1 z}$$

$$\vec{H_r}(z) = \frac{1}{\eta_1} \hat{a_{nr}} \times \vec{E_r}(z)$$

$$= \frac{1}{\eta_1} (-\hat{a_z}) \times \vec{E_r}(z)$$

$$\vec{H_r}(z) = -\hat{a_y} \frac{1}{\eta_1} E_{ro} e^{+j\beta_1 z}$$

Total wave in medium 1

$$\vec{E_1}(z) = \vec{E_i}(z) + \vec{E_r}(z)$$

$$= \hat{a_x} (E_{io} e^{-j\beta_1 z} + E_{ro} e^{+j\beta_1 z})$$

Continuity of tangential component of the E field at the boundary $z = 0$

$$\vec{E_1}(0) = \hat{a_x} (E_{io} + E_{ro}) = E_2(0) = 0$$

$$\Rightarrow E_{ro} = -E_{io}$$

$$\therefore \vec{E_1}(z) = \hat{a_x} E_{io} (e^{-j\beta_1 z} - e^{+j\beta_1 z}) = -\hat{a_x} j 2 E_{io} \sin \beta_1 z$$

$$\therefore \vec{H_1}(z) = \vec{H_i}(z) + \vec{H_r}(z)$$

$$= \hat{a_y} 2 \frac{E_{io}}{\eta_1} \cos \beta_1 z$$

$\therefore \vec{E}_1(z) = \vec{a_x}\ E_{i0}\left(e^{-j\beta_1 z} - e^{+j\beta_1 z}\right)$

$= -\hat{a}_x\ j2E_{i0} \sin\beta_1 z$

$\therefore \vec{H}_1(z) = \vec{H}_i(z) + \vec{H}_r(z)$

$= \hat{a}_y\ 2\frac{E_{i0}}{\eta_1} \cos\beta_1 z$

the space-time behavior of the total field in medium 1

$\vec{E}_1(z,t) = Re\left[\vec{E}_1(z)\ e^{j\omega t}\right] = \hat{a}_x 2E_{i0} \sin\beta_1 z \sin\omega t$

$\vec{H}_1(z,t) = Re\left[\vec{H}_1(z)\ e^{j\omega t}\right] = \hat{a}_y\ 2\frac{E_{i0}}{\eta_1} \cos\beta_1 z \cos\omega t$

zeros of $\vec{E}_1(z,t)$ }
maxima of $\vec{H}_1(z,t)$ } occur at $\beta_1 z = -n\pi$ (or)

$z = -n\frac{\lambda}{2}, \quad n = 0, 1, 2, \ldots$

maxima of $\vec{E}_1(z,t)$ }
zeros of $\vec{H}_1(z,t)$ } occur at $\beta_1 z = -(2n+1)\frac{\pi}{2}$

(or) $z = -(2n+1)\frac{\lambda}{4}, \quad n = 0, 1, 2, \ldots$

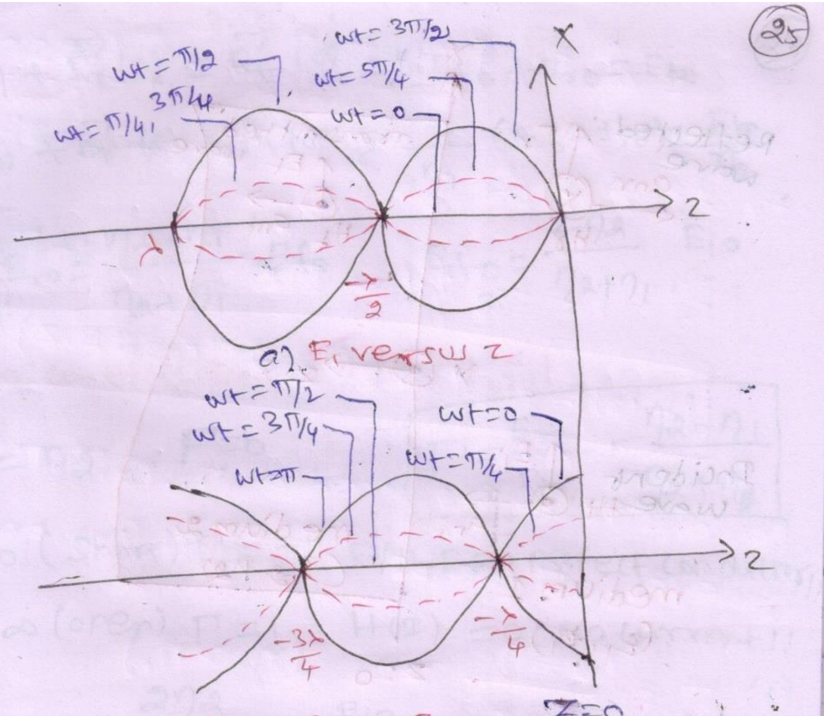
$\vec{E}_1(z,t) = Re\left[\vec{E}_1(z)\ e^{j\omega t}\right]$

$= \hat{a}_x\ 2E_{i0} \sin\beta_1 z \sin\omega t$

$\vec{H}_1(z,t) = Re\left[\vec{H}_1(z)\ e^{j\omega t}\right]$

$= \hat{a}_y\ 2 \cdot \frac{E_{i0}}{\eta_1} \cos\beta_1 z \cos\omega t$

The total wave in medium 1 is not a travel[ling] wave. Standing wave.



a) E_1 versus z

b) H_1 versus z

Fig: standing waves of $E_1 = a_x E_1$ and $H_1 = a_y H_1$ for several values of wt

Note following three points

(i) E vanishes on the conducting Boundary

(ii) $\vec{H}$ a maximum on the conducting boundary

(iii) the standing waves of $\vec{E_1}$ and $\vec{H_1}$ are in time quadrature (90° phase diffeence

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Normal incidence at a plane dielectric boundary

$\sigma_1 = \sigma_2 = 0, \quad \varepsilon_1 \neq \varepsilon_2$

Incident wave (inside medium 1)

$\vec{E_i}(z) = \hat{a}_x E_{i0} e^{-j\beta_1 z}$

$\vec{H_i}(z) = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$

✓ minimum value of $|\vec{E}_1(z)|$ is $E_{i0}(1+\Gamma)$. (29)

at $z_{max} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}$, $n=0,1,2\ldots$

$$\boxed{S = \frac{|E|_{max}}{|E|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$ Standing wave Ratio (SWR)

$$\boxed{|\Gamma| = \frac{S-1}{S+1}}$$ $(-1 \le \Gamma \le 1, 1 \le S \le \infty)$

if $\Gamma = 0, S = 1$, No reflection, full powe transmission

if $\Gamma = 1, S = \infty$, Total reflection, no powe transmission