EC3452-ELECTROMAGNETIC FIELDS

(Regulation 2021)

UNIT IV – TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Part A - Two Marks

1. Define a wave.

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location then the group of phenomena constitutes a wave.

2. Mention the properties of uniform plane wave.

i) At every point in space, the electric field E and magnetic field H are perpendicular to each other.

ii) The fields vary harmonically with time and at the same frequency everywhere in space.

3. Define intrinsic impedance or characteristic impedance.

It is the ratio of electric field to magnetic field or It is the ratio of square root of permeability to permittivity of medium.

4. Give the characteristic impedance of free space.

377ohms.

5. Define propagation constant.

Propagation constant is a complex number $\gamma = \alpha + j\beta$ where γ is propagation constant.

6. Define skin depth

It is defined as that depth in which the wave has been attenuated to 1/e or approximately 37% of its original value.

7. Define Poynting vector.

The pointing vector is defined as rate of flow of energy of a wave as it propagates. P =E X H

8. State Poynting's Theorem.

The net power flowing out of a given volume is equal to the time rate of decrease of the energy stored within the volume conduction losses.

9. Give the difficulties in FDM.

FDM is difficult to apply for problems involving irregular boundaries and non-homogeneous material properties.

10. Explain the steps in finite element method.
i) Discrimination of the solution region into elements.
ii) Generation of equations for fields at each element.
iii) Assembly of all elements.
iv) Solution of the resulting system.

11. State Maxwell's fourth equation.

The net magnetic flux emerging through any closed surface is zero.

12. State Maxwell's Third equation

The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

13. State the principle of superposition of fields.

The total electric field at a point is the algebraic sum of the individual electric field at that point.

14. Define loss tangent.

Loss tangent is the ratio of the magnitude of conduction current density to displacement current density of the medium.

15. Define reflection coefficient.

Reflection coefficient is defined as the ratio of the magnitude of the reflected field to that of the incident field.

16. Define transmission coefficient.
Transmission coefficient is defined as the ratio of the magnitude of the transmitted field to that of incident field.

17. What will happen when the wave is incident obliquely over dielectric – dielectric boundary?

When a plane wave is incident obliquely on the surface of a perfect dielectric part of the energy is transmitted and part of it is reflected .But in this case the transmitted wave will be refracted, that is the direction of propagation is altered.

18. What are uniform plane waves?

Electromagnetic waves which consist of electric and magnetic fields that are perpendicular to each other and to the direction of propagation and are uniform in plane perpendicular to the direction of propagation are known as uniform plane waves.

PART-B

1) Explain the Maxwel equation I &II.

Moving conductor in time varying magnetic field

If a moving conductor carrying current I is placed in a time varying magnetic field, then the induced emf is the sum of both transformer emf & motional emf.

$$V = \oint_l E \cdot dl = \text{Transformer emf} + \text{motional emf}$$

$$V = -\int_S \frac{\partial B}{\partial t} \cdot ds + \oint_l (v \times B) dl$$

Differential & integral form of Maxwell's equations

Maxwell's equation I

From Ampere's circuital law:-

It states that line integral of magnetic field intensity H on any closed path is equal to current enclosed by that path.

$$\oint H \cdot dl = I = \int_S J \cdot ds$$

current involves both conduction current & displacement current.

A current through resistive element is called condition current where as current through capacitive element is called displacement current.

current through a conductor of resistance R is

But $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$ $\sigma = \frac{1}{\rho}$

σ – conductivity

$A \rightarrow$ Area of cross section ρ – Resistivity (11)

$$I_c = \frac{V \sigma A}{l}$$

if E is electric field then $V = El$

$$I_c = \frac{E l \sigma A}{l} = \sigma E A$$

$$\frac{I_c}{A} = \sigma E = J_c$$

current through a capacitor is

$$I_D = dQ/dt$$

$$Q = CV$$

$$I_D = C\frac{dV}{dt}$$

W.K.T

$$C = \varepsilon A/d$$

ε – permittivity of medium

A – Area of $\parallel^{el}$ plate capacitor

d – Distance b/w 2 plates

$$I_D = \frac{\varepsilon A}{d}\frac{dV}{dt}$$

$$V = Ed$$

$$I_D = \frac{\varepsilon A}{d} d \frac{dE}{dt} \qquad I_D = \varepsilon A \frac{dE}{dt}$$

$$\frac{I_D}{A} = \varepsilon \frac{\partial E}{\partial t} = \frac{dD}{dt}$$ (12)

$$J_D = \frac{\partial D}{\partial t}$$

$$\oint H \cdot dl = \iint (J_c + J_D)\, ds$$

$$\oint H \cdot dl = \iint \left(\sigma E + \frac{\partial D}{\partial t}\right) ds$$

$$\oint H \cdot dl = \iint \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) ds$$

$$\boxed{\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds} \longrightarrow ①$$

eqn ① is Maxwell's equations I in integral form from ampere's circuital law

By applying stoke's theorem

$$\oint H \cdot dl = \iint (\nabla \times H) \cdot ds \longrightarrow ②$$

Comparing eqn ① & ②

$$\nabla \times H = J + \frac{\partial D}{\partial t} \longrightarrow ③$$

eqn ③ is Maxwell's eqn I in point or differential form

2) Explain the Maxwel Equations III & IV.

Maxwell's equation III

Electric Gauss law: It states that electric flux passing through any closed surface is equal to charge enclosed by that surface.

$$\psi = Q$$

$$\iint D.ds = Q \quad (or) \quad \iiint \rho_v \, dv = Q$$

$$\iint D.ds = \iiint \rho_v \, dv \rightarrow (7)$$

eqn (7) is maxwell's eqn III in integral form. By applying Divergence theorem

$$\iint D.ds = \iiint \nabla.D dV \rightarrow (8)$$

Comparing eqn (7) & (8)

$$\iiint \nabla.D dv = \iiint \rho_v \, dv$$

$$\boxed{\nabla.D = \rho_v = \rho} \rightarrow (9)$$

eqn (9) is maxwell's equation III in point or differential form

Maxwell's equation IV (15)

Magnetic Gauss law: It states that total magnetic flux through any closed surface is equal to zero

$$\phi = 0$$

$$\iint B \cdot ds = 0 \rightarrow (10)$$

eqn (10) is maxwell's equations IV in integral form.

By applying Divergence theorem

$$\iint B \cdot ds = \iiint \nabla \cdot B \, dv \rightarrow (11)$$

comparing eqn (10) & (11)

$$\iiint \nabla B \, dv = 0$$

$$\boxed{\nabla \cdot B = 0} \rightarrow (12)$$

eqn (12) is Maxwell's equation IV in point or differential form

*) If $\vec{D} = 10x \vec{a_x} - 4y \vec{a_y} + Kz \vec{a_z}$ $\mu C/m^2$ & $\vec{B} = 2\vec{a_y}$ mT. Find the Value of K to satisfy the Maxwell's equations for region $\sigma = 0$, $\rho_v = 0$

Sol $\vec{D} = 10x \vec{a_x} - 4y \vec{a_y} + Kz \vec{a_z}$ $\mu C/m^2$

$\vec{B} = 2\vec{a_y}$ mT, $\sigma = 0$, $\rho_v = 0$

$$\left(\frac{d}{dx}\vec{a_x}+\frac{d}{dy}\vec{a_y}+\frac{d}{dz}\vec{a_z}\right)\cdot\left(10x\vec{a_x}-4y\vec{a_y}+kz\vec{a_z}\right)=$$

16

$$\frac{d}{dx}(10x)-\frac{d}{dy}(4y)+\frac{d}{dz}(kz)=0$$

$$10-4+k=0$$

$$k=-6\mu C/m^2$$

*) if the magnetic field $\vec{H}=(3x\cos\beta+6y\sin\alpha)\vec{a_z}$. Find current density $\vec{J}$ if fields are invarient with time.

$$\vec{H}=(3x\cos\beta+6y\sin\alpha)\vec{a_z}$$

From Maxwell's 2nd eqn

$$\nabla\times H=\vec{J}+\frac{d\vec{D}}{dt}$$

The fields are invarient with time so $\boxed{\frac{dD}{dt}=0}$

$$\nabla\times H=\vec{J}$$

$$\vec{J}=\begin{vmatrix}\vec{a_x} & \vec{a_y} & \vec{a_z}\\ d/dx & d/dy & d/dz\\ 0 & 0 & 3x\cos\beta+6y\sin\alpha\end{vmatrix}$$

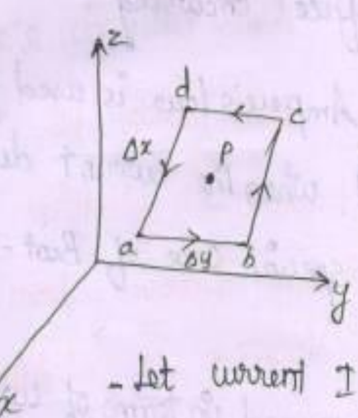
$$=\vec{a_x}\left[\frac{d}{dy}(3x\cos\beta+6y\sin\alpha)\right]-\vec{a_y}\left[\frac{d}{dx}(3x\cos\beta+6y\sin\alpha)\right]+\vec{a_z}(0-0)$$

$$\vec{J}=6\sin\alpha\,\vec{a_x}-3\cos\beta\,\vec{a_y}\ A/m^2$$

The above equation is one of the Maxwell's equations applicable to static magnetic fields & is known as the point form or differential form of Ampere's law. (14)

- The differential form of Ampere's circuital law states that the curl of the magnetic field intensity H is equal to the conduction current density $\nabla \times H = J$

Proof



- consider a different surface element having sides $\Delta x, \Delta y$ in the xy plane centered at point P

- Let current I with current density $\vec{J}$ flows through the closed path along the z-direction. The magnetic field intensity at point P in rectangular co-ordinate is

$$\vec{H_0} = H_{x_0}\vec{a_x} + H_{y_0}\vec{a_y} + H_{z_0}\vec{a_z}$$

while the total current density is given by

$$\vec{J} = J_x\vec{a_x} + J_y\vec{a_y} + J_z\vec{a_z}$$

From Ampere's circuital law

$$\oint \vec{H}\cdot\vec{dl} = I$$

Where I is the net current flowing in the closed path a-b-c-d-a where J_z is the z-component of current density written as

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z \quad (16)$$

$||^{ly}$ the x & y components of the current density are

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\vec{J} = J_x \vec{a_x} + J_y \vec{a_y} + J_z \vec{a_z}$$

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a_y} = \vec{J}$$

In matrix form

$$\vec{J} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

The above determinant is known as curl $\vec{H}$, represented by $\Delta \times \vec{H}$ i.e., the cross product of del operator & $\vec{H}$.

Hence $\nabla \times \vec{H} = \vec{J}$

3)Explain the wave equation for conducting medium

Electromagnetic wave equation

26

Wave equation for conducting medium

The maxwell's equation from Faraday's law in point form is given by

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

Taking curl on both sides

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}(\nabla \times H) \rightarrow (2)$$

Maxwell's eqn from ampere's law in point form is given by

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t} \rightarrow (3)$$

Sub eqn (3) in eqn (2)

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$= -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (4)$$

But from vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E \rightarrow (5)$$

$$\nabla \cdot E = \frac{\nabla \cdot D}{\varepsilon}$$

Since there is no net charge within the conductor, the charge density $\rho = 0$

$$\nabla \cdot D = 0$$

eqn (5) becomes

(27)

$$\nabla \times \nabla \times E = -\nabla^2 E \rightarrow (6)$$

Comparing eqn (4) & (6)

$$-\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (7)$$

$$\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \rightarrow (8)$$

This is the wave eqn in terms of electric field E.

The wave equation in terms of magnetic field H is obtained in a similar manner as follows.

The Maxwell's eqn from Ampere's law in point form is given by

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t} \rightarrow (9)$$

Taking curl on both sides

$$\nabla \times \nabla \times H = \sigma\, \nabla \times E + \varepsilon \frac{\partial}{\partial t} (\nabla \times E) \rightarrow (10)$$

But from Faraday's law

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (11)$$

Sub eqn (11) in eqn (10)

$$\ldots \mu\sigma \frac{\partial H}{\ldots} - \mu\varepsilon \frac{\partial^2 H}{\ldots} \rightarrow (12)$$

From Vector identity

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H \rightarrow (13)$$

$$\nabla \cdot B = \mu \nabla \cdot H = 0$$

eqn (13) becomes

$$\nabla \times \nabla \times H = -\nabla^2 H \rightarrow (14)$$

comparing eqn (12) & (14)

$$-\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0}$$

This is the wave eqn in terms of magnetic field H.

Wave equation for free space

For free space the conductivity of the med is zero (i.e $\sigma = 0$) & there is no charge contained in it (i.e $\rho = 0$)

The Maxwell's equation from Faraday's law for free space in point form is

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

Taking curl on both sides

$$\ldots \mu \frac{\partial}{\partial t}(\nabla \times H) \rightarrow (2)$$

4) Explain the wave Equation for free Space:

Wave equation for free space 31

$\sigma = 0$ & $\rho = 0$

The Maxwell's eqn from faraday's law for free space in point form is

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \rightarrow (1)$$

Taking curl On both sides

$$\nabla \times \nabla \times E = -\mu \frac{d}{dt} (\nabla \times H) \rightarrow (2)$$

The Maxwell's equation from Ampere's law for free Space in point form is

$$\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \rightarrow (3)$$

Sub eqn (3) in eqn (2)

$$\nabla \times \nabla \times E = -\mu \frac{d}{dt} \left(\varepsilon \frac{\partial E}{\partial t} \right)$$

$$= -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (4)$$

From Vector identity

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\nabla \cdot E = \frac{\nabla \cdot D}{\varepsilon} = \frac{\rho}{\varepsilon} = 0$$

$$\nabla \times \nabla \times E = -\nabla^2 E \rightarrow (5)$$

Compare eqn (4) & eqn (5)

$\mu \varepsilon \, \partial^2 E / \partial t^2$

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \rightarrow (6)$$

This is the wave equation for free space interms of electric field.

The wave equation for free space interms of magnetic field H is obtained in a similar manner follows.

The Maxwell's eqn from Ampere's law for free in point form is given by

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \rightarrow (7)$$

Taking curl On both sides

$$\nabla \times \nabla \times H = \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \rightarrow (8)$$

The Maxwell's eqn from faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \rightarrow (9)$$

Sub eqn (9) in eqn (8)

$$\nabla \times \nabla \times H = -\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial t}\right)$$

$$\nabla \times \nabla \times H = -\mu\varepsilon \frac{\partial^2 H}{\partial t^2} \rightarrow (10)$$

From Vector identity

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H$$

$$\nabla \cdot H = \frac{1}{\mu} \nabla \cdot B = 0$$

$$\nabla \times \nabla \times H = -\nabla^2 H \longrightarrow (11)$$

Compare eqn (10) & (11)

$$-\nabla^2 H = -\mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H = \mu \varepsilon\, \partial^2 H / \partial t^2$$

$$\nabla^2 H - \mu \varepsilon \frac{\partial^2 H}{\partial t^2} = 0 \longrightarrow (12)$$

This is the wave eqn for free space interms of H

for free space $\mu_r = 1$ & $\varepsilon_r = 1$

$$\nabla^2 H - \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

or

$$\boxed{\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}}$$

Solution of wave equation

considering a plane wave propagating in x direction.

The wave eqn for free space is

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

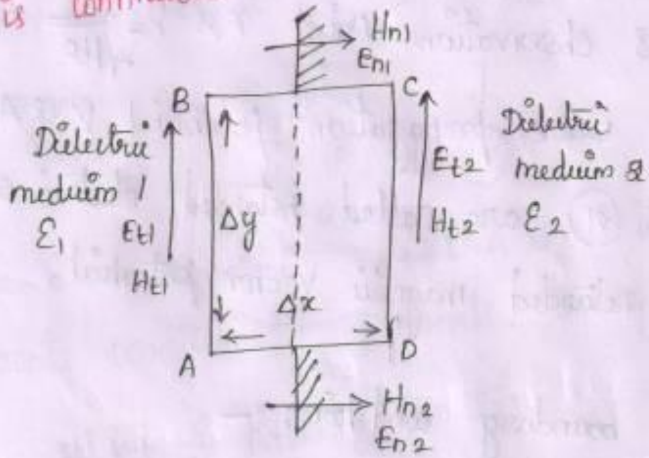
The general solution of this differential equation is of the form

$$E = f_1(x - V_0 t) + f_2(x + V_0 t)$$

5) Explain the Boundary conditions of Normal component.

3. The normal component of electric flux density D is continuous if there is no surface charge density, or D is discontinuous by an amount equal to surface charge density.

4. The normal component of magnetic flux density B is continuous at the surface of discontinuity.



consider a rectangle of length Δy & width Δx at the boundary of 2 dielectric media.

$$V = \oint E \cdot dl = 0$$

Apply this to the rectangular path ABCD in which AB just inside the medium 2

$$\oint E \cdot dl = E_{t1} \Delta y + E_{n1} \Delta x - E_{t2} \Delta y - E_{n2} \Delta x$$

E_{t1} & E_{t2} are tangential component of E along the path AB & CD

E_{n1} & E_{n2} are normal component of E along the path

$D_{n1}\,ds - D_{n2}\,ds = 0$

$D_{n1} = D_{n2}$

The normal component of D is Continuous if there is no Surface charge density

If the charges are enclosed by pill box $\Delta h \to 0$

$\int D \cdot ds = Q$

$D_{n1}\,ds - D_{n2}\,ds = Q$

$D_{n1} - D_{n2} = \frac{Q}{ds} = \rho_s$

$D_{n1} - D_{n2} = \rho_s$

The normal component of D is discontinuous across the boundary by the amount of surface charge density. The integral form of Maxwell's 4th eqn is

$\iint B \cdot ds = 0$

Apply to the pill box at the boundary

$B_{n1}\,ds - B_{n2}\,ds = 0$

$B_{n1} = B_{n2}$

The normal component of magnetic flux density B is Continuous across the boundary.

$$D_{n1}\, ds - D_{n2}\, ds = 0$$

$$D_{n1} = D_{n2}$$

The normal component of D is Continuous if there is no Surface charge density

If the charges are enclosed by pill box $\Delta h \to 0$

$$\int D . ds = Q$$

$$D_{n1}\, ds - D_{n2}\, ds = Q$$

$$D_{n1} - D_{n2} = \frac{Q}{ds} = \rho_s$$

$$D_{n1} - D_{n2} = \rho_s$$

The normal component of D is discontinuous across the boundary by the amount of surface charge density. The integral form of Maxwell's 4th eqn is

$$\iint B . ds = 0$$

Apply to the pill box at the boundary

$$B_{n1}\, ds - B_{n2}\, ds = 0$$

$$B_{n1} = B_{n2}$$

The normal component of magnetic flux density B is Continuous across the boundary.