SRI MUTHUKUMARAN INSTITUTE OF TECHNOLOGY
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EC3452-ELECTROMAGNETIC FIELDS
(Regulation 2021)

UNIT III – MAGNETOSTATICS

Part A - Two Marks

1. What is Magnetic Field?

The region around a magnet within which influence of the magnet can be experienced is called Magnetic Field.

2. What are Magnetic Lines of Force?

The existence of Magnetic Field can be experienced with the help of compass field. Such a field is represented by imaginary lines around the magnet which are called Magnetic Lines of Force.

3. State Stoke Theorem.

The line integral of F around a closed path L is equal to the integral of curl of F over the open surface S enclosed by the closed path L.

4. Define scalar magnetic Potential.

The scalar magnetic potential V_m can be defined for source free region where J i.e. current density is zero.

5. What is the fundamental difference between static electric and magnetic field lines?

There is a fundamental difference between static electric and magnetic field lines. The tubes of electric flux originate and terminate on charges, whereas magnetic flux tubes are continuous.

6. State Kirchoff's Flux law.

It states that the total magnetic flux arriving at any junction in a magnetic circuit is equal to the magnetic flux leaving that junction. Using this law, parallel magnetic circuits can be easily analyzed.

7. State Kirchoff's MMF law.

Kirchoff's MMF law states that the resultant MMF around a closed magnetic circuit is equal to the algebraic sum of products of flux and reluctance of each part of the closed circuit.

8. What is Magnetization?

The field produced due to the movement of bound charges is called Magnetization represented by *M*.

9. State Biot Savart Law.

The Biot Savart law states that, the magnetic field intensity *dH* produced at a point p due to a differential current element IdL is,

1) Proportional to the product of the current I and differential length dL.
2) The sine of the angle between the element and the line joining point p to the element and
3) Inversely proportional to the square of the distance R between point p and the element.

10. Describe what are the sources of electric field and magnetic field?

Stationary charges produce electric field that are constant in time, hence the term electrostatics. Moving charges produce magnetic fields hence the term magnetostatics.

11. Define Magnetic flux density.

The total magnetic lines of force i.e. magnetic flux crossing a unit area in a plane at right angles to the direction of flux is called magnetic flux density. Unit Wb/m^2.

12. State Ampere's circuital law.

The line integral of magnetic field intensity H around a closed path is exactly equal to the direct current enclosed by that path.

13. Define Magnetic field Intensity.

Magnetic Field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole of one Weber strength, when placed at that point. Unit: N/Wb.

PART-B

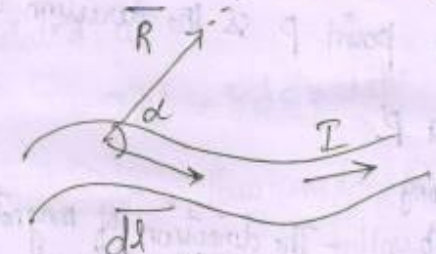
1) Explain the Biot savart law with relevant examples.

BIOT- SAVART'S LAW (1)

It states that the differential magnetic field intensity dH produced at a point p by the differential current element I dl is proportional to the product I dl & the sine of the angle α b/w the element & the line joining p to the element & is inversely proportional to the square of the distance R b/w p & the element.

Magnetic field dH at a point p due to current element I dl

$\odot P$ dH out of the page



$P' \otimes$ dH into the page

Fig 1

$$dH = \frac{I\,dl \sin\alpha}{R^2} \rightarrow (1)$$

$$dH = \frac{K\,I\,dl \sin\alpha}{R^2} \rightarrow (2)$$

$K = 1/4\pi$. So eqn (2) becomes

Using the magnitude of crossproduct

i.e $|dl \times R| = dl\, R \sin\alpha$ in the above eqn, the magnitude of differential magnetic field intensity becomes

$$dH = \frac{I\, dl \sin\alpha}{4\pi R^2} \rightarrow (4)$$

eqn (4) put in vector form as

$$\vec{dH} = \frac{I\,\vec{dl} \times \vec{a_R}}{4\pi R^2} = \frac{I\,\vec{dl} \times \vec{R}}{4\pi R^3} \rightarrow (5)$$

$R = |R|$ & $\vec{a_R} = \vec{R}/R$

The conventional representation of magnetic field intensity in fig 1 in which the direction of dH is out of the page at point P & the direction of dH is into the page point P'.

- The direction of magnetic field dH can be determined by the right hand rule. Here, the thumb points in the direction of current & the remaining 4 fingers surrounding the wire represents the direction of magnetic field

The direction of dH can also be determined using the RH screw rule. Here, the tip of the screw moving downwards points in the direction of current I & the direction of dH shown in fig.

We can have different charge configurations. We can have different current distributions: lines current, Surface current & Volume current. If we define K as the Surface current density in amperes/meter & J as the volume current density in amperes/m^2, the Source elements are related as

$$I\,dl = K\,ds = J\,dv$$

Biot-Savart's law can be expressed in terms of distributed current Source as follows

$$H = \int_l \frac{I\,\vec{dl} \times \vec{a_R}}{4\pi R^2} \quad \text{(line current)}$$

$$H = \int_S \frac{K\,\vec{ds} \times \vec{a_R}}{4\pi R^2} \quad \text{(Surface current)}$$

$$H = \int_V \frac{J\,\vec{dv} \times \vec{a_R}}{4\pi R^2} \quad \text{(Volume current)}$$

is an unit vector pointing from the differential

Magnetic field intensity at O due to the entire circular wire is (11)

$$\int dH = \int \frac{I\,dl}{4\pi a^2} \vec{a_z}$$

$$H = \frac{I}{4\pi a^2} \int dl\, \vec{a_z}$$

$\oint dl =$ circumference of the wire $= 2\pi a$

$$H = \frac{I}{4\pi a^2} [2\pi a] \vec{a_z}$$

$$\boxed{H = \frac{I}{2a} \vec{a_z}}$$

*) A wire carrying a current of 8 A is formed into a circular loop. If the magnetic field intensity at the centre of the loop is 40 A/m.

Sol $I = 8A \quad H = 40\ A/m$

Magnetic field intensity H at the centre of the circular loop carrying current I is

$$H = \frac{I}{2a} \vec{a_z}$$

For N turns, the magnetic field intensity

$$H = \frac{NI}{2a}$$

(i) The radius of the loop for N=1 turn is

$$a = \frac{I}{2H} = \frac{8}{2 \times 40} = 0.1\,m$$

(ii) The radius of the loop for N=10 turn is

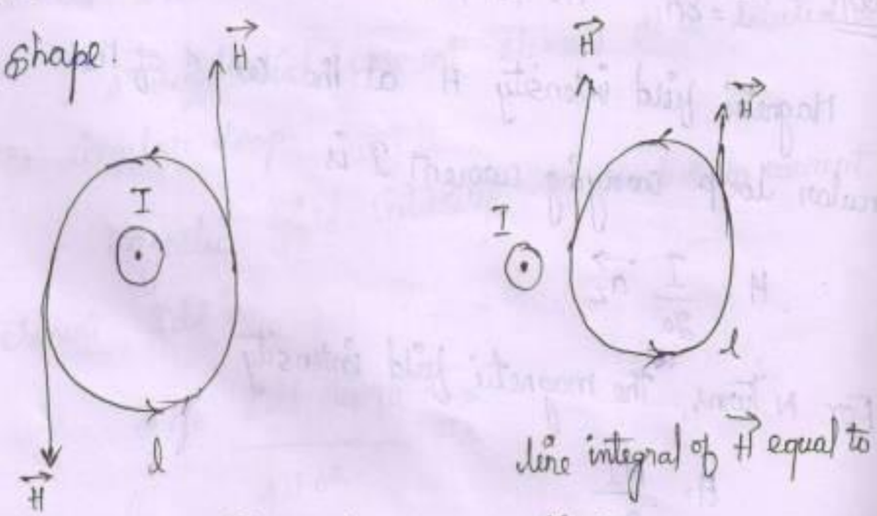
$$a = \frac{NI}{2H} = \frac{10 \times 8}{2 \times 40} = 1\,m$$

Ampere's circuital law

It states that the line integral of the magnetic field intensity $\vec{H}$ around a closed path l is equal to the total current enclosed by the path. In other words the circulation of H equals I.

$$\oint H \cdot dl = I$$

where I is the total current enclosed by the path.
The above equation is called integral form of Ampere's law.
The current can be carried by a conductor of any shape.



line integral of $\vec{H}$ equals to I

line integral of $\vec{H}$ equal to zero

2) Explain the Ampere's circuit law.

Point form of Ampere's circuital law 13

- In electrostatics, Gauss's law is used to determine the electric field intensity & flux density in a region having symmetrical charge distribution.

- The determination of field depends on the proper choice of the Gaussian surface enclosing the charges.

- ||'ly in magnetostatics, Ampere's law is used to determine the magnetic field when the current distribution is symmetrical. It is a special case of Biot-Savart's law.

The current can be expressed in terms of current density J as

$$I = \int_S J \cdot ds \rightarrow (1)$$

$$\int_l H\, dl = \int_S J \cdot ds \rightarrow (2)$$

The line integral on the LHS of the above equation can be converted to surface integral by using Stoke's theorem

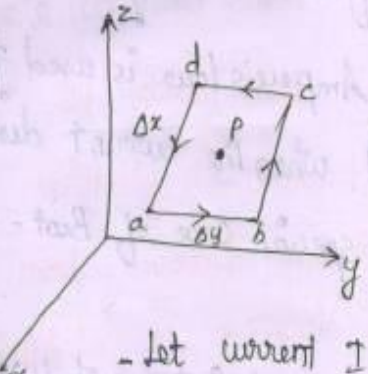
$$\int_S (\nabla \times H)\, ds = \int_S J \cdot ds \rightarrow (3)$$

Comparing the surface integrals on both sides of the

The above equation is one of the Maxwell's equations applicable to static magnetic fields & is known as the point form or differential form of Ampere's law. (14)

- The differential form of Ampere's circuital law states that the curl of the magnetic field intensity H is equal to the conduction current density $\nabla \times H = J$

Proof



- consider a different surface element having sides $\Delta x, \Delta y$ in the xy plane centered at point P

- Let current I with current density $\vec{J}$ flows through the closed path along the z-direction. The magnetic field intensity at point P in rectangular co-ordinate is

$$\vec{H_0} = H_{x_0}\vec{a_x} + H_{y_0}\vec{a_y} + H_{z_0}\vec{a_z}$$

while the total current density is given by

$$\vec{J} = J_x\vec{a_x} + J_y\vec{a_y} + J_z\vec{a_z}$$

From Ampere's circuital law

$$\oint \vec{H}\cdot\vec{dl} = I$$

where I is the net current flowing in the closed path a-b-c-d-a where J_z is the z-component of current density written as

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z \qquad (16)$$

$\|^{ly}$ the x & y components of the current density are

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\vec{J} = J_x \vec{a_x} + J_y \vec{a_y} + J_z \vec{a_z}$$

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a_y} = \vec{J}$$

In matrix form

$$\vec{J} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

The above determinant is known as curl $\vec{H}$, represented by $\Delta \times \vec{H}$ i.e., the cross product of del operator & $\vec{H}$.

Hence $\nabla \times \vec{H} = \vec{J}$

3) Explain the Sclar and vector magnetic field.

Scalar and Vector magnetic potentials

- Electrostatic field problems were simplified by relating the electric potential V to the electric field intensity $\vec{E}$ ($\vec{E} = -\nabla V$). 11[ly] define a potential associated with magnetic field B. The magnetic potential could be scalar V_m or Vector A. (23)

- The negative gradient of scalar magnetic potential V_m gives the magnetic field intensity as represented below.

$$\vec{H} = -\nabla V_m \rightarrow (1)$$

The unit of scalar magnetic potential is ampere.

From point form of Ampere's law

$$\vec{J} = \nabla \times \vec{H} \rightarrow (2)$$

Sub eqn (1) in eqn (2)

$$\vec{J} = \nabla \times (-\nabla V_m) \rightarrow (3)$$

As per Vector identity rules, the curl of the gradient of any Scalar field should be zero.

i.e. $\nabla \times (\nabla V) = 0$. Since the above equation involves curl of the gradient of the scalar magnetic potential, the current density will be zero throughout the region in which the scalar is defined ∴ the scalar magnetic potential V_m gives in equation (1) is applicable only if $\vec{J} = 0$ i.e V_m is defined in a region where $\vec{J} = 0$

The scalar potential also satisfies laplace equation. In free space

(24)

$$\nabla \cdot \vec{B} = \mu_0 \nabla \cdot \vec{H} = 0 \longrightarrow (4)$$

& hence

$$\mu_0 \nabla \cdot (-\nabla V_m) = 0$$

$$\nabla^2 V_m = 0 \quad (\text{when } \vec{J} = 0) \rightarrow (5)$$

The above equation shows that the laplace's equation in static magnetic fields is applicable only in a current free region where $\vec{J}=0$ similar to scalar magnetic potential V_m.

As the divergence of $\vec{B}$ is zero, the magnetic flux density is always solenoidal. A vector whose divergence is zero can be expressed in terms of curl of another vector quantity as

$$\vec{B} = \nabla \times \vec{A} \longrightarrow (6)$$

$\vec{A}$ is called the vector magnetic potential expressed in weber/meter (Wb/m)

Taking curl on both sides

$$\nabla \times \vec{B} = \nabla \times \nabla \times \vec{A}$$

By the identity $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

$$\nabla(\nabla\times\vec{A}) - \nabla^2\vec{A} = \mu J$$

For the situation of steady direct currents (25)

$$\nabla\times\vec{A} = 0$$

So that $-\nabla^2\vec{A} = \mu\vec{J}$ (for direct currents only) → (7)

Expanding both sides

$$\nabla^2 A_x \vec{a_x} + \nabla^2 A_x \vec{a_y} + \nabla^2 A_z \vec{a_z} = \mu\left[J_x\vec{a_x} + J_y\vec{a_y} + J_z\vec{a_z}\right]$$

Equating

$$\left.\begin{aligned} \nabla^2 A_x &= -\mu J_x \\ \nabla^2 A_y &= -\mu J_y \\ \nabla^2 A_z &= -\mu J_z \end{aligned}\right\} \longrightarrow (8)$$

Each of the above 3 equations is of the same form as the poisson equation

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

where V is the electrostatic potential

$$V = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho\, dv}{r}$$

In the relations (8) we have on the RHS J term instead of ρ, & μ instead of $1/\varepsilon_0$ so that the solution can be written by inspection as follows

$$A_x = \frac{\mu}{4\pi}\int_v \left(\frac{J_x}{r}\right) dv \qquad A_y = \frac{\mu}{4\pi}\int_v \left(\frac{J_y}{r}\right) dv$$

4) Problem:

*) Given $\vec{A} = (y\cos ax)\vec{a_x} + (y+e^x)\vec{a_z}$ determine $\nabla \times \vec{A}$ at the Origin.

(28)

Sol

$$\vec{A} = (y\cos ax)\vec{a_x} + (y+e^x)\vec{a_z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\cos ax & 0 & y+e^x \end{vmatrix}$$

$$= \vec{a_x} - e^x\vec{a_y} - \cos ax\,\vec{a_z}$$

At Origin (0,0,0)

$$\nabla \times \vec{A} = \vec{a_x} - \vec{a_y} - \vec{a_z}$$

Derivation of steady magnetic field laws

Biot-Savart's law & Ampere's law the two important laws used in static magnetic potential.

The expression for vector magnetic potential $\vec{A}$ due to line current is

$$\vec{A} = \frac{\mu_0}{4\pi}\oint_{l'} I\frac{\vec{dl'}}{R}$$

Since $\vec{B} = \nabla \times \vec{A}$ the magnetic flux density can be written as

$$\vec{B} = \nabla \times \left[\frac{\mu_0}{4\pi}\oint_{l'} I\frac{\vec{dl'}}{R}\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\oint_{l'} \nabla \times \frac{1}{R}\,\overrightarrow{dl'}\right] \rightarrow (1)$$

Considering the vector identity

$$\nabla \times (f\vec{F}) = f\nabla \times \vec{F} + (\nabla f) \times \vec{F}$$

Taking $\vec{F} = \overrightarrow{dl'}$ & $f = \frac{1}{R}$ eqn (1) can be changed to

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_{l'}\left[\frac{1}{R}(\nabla \times dl') + (\nabla \frac{1}{R}) \times \overrightarrow{dl'}\right] \rightarrow (2)$$

∇ operates w.r.t (x,y,z) & $\overrightarrow{dl'}$ is a function of (x',y',z') the curl of $\overrightarrow{dl'}$ becomes zero i.e $\nabla \times \overrightarrow{dl'} = 0$. Hence the above eqn becomes

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_{l'}\left[\left(\nabla \frac{1}{R}\right) \times \overrightarrow{dl'}\right] \rightarrow (3)$$

The magnitude of distance vector is

$$R = |\vec{R}| = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

& inverse

$$\frac{1}{R} = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{-1/2}$$

$$\therefore \nabla\left(\frac{1}{R}\right) = \frac{(x-x')\vec{a_x} + (y-y')\vec{a_y} + (z-z')\vec{a_z}}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$\boxed{\nabla(1/R) = -\vec{a_R}/R^2} \rightarrow (4)$$

... source point to field point

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \left[\left(-\frac{\vec{a_R}}{R^2} \right) \times \vec{dl} \right] \rightarrow (5)$$

The -ve sign in the above equation can be removed by interchanging the terms of the vector product.

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint_l \left[\frac{\vec{dl} \times \vec{a_R}}{R^2} \right]$$

Since $\vec{B} = \mu_0 \vec{H}$ the magnetic field intensity is written as

$$\vec{H} = \frac{I}{4\pi} \oint_l \frac{\vec{dl} \times \vec{a_R}}{R^2} \rightarrow (6)$$

Here the current I flow along the closed path l.

Hence the magnetic field intensity derived in the above equation using the vector magnetic potential is known as Biot - Savart's law.

Stoke's theorem is given by

$$\oint_l \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}$$

$$= \frac{1}{\mu_0} \int_S (\nabla \times \vec{B}) \cdot \vec{ds}$$

$$= \frac{1}{\mu_0} \int_S (\nabla \times \nabla \times A) \vec{ds} \rightarrow (7)$$

By using the vector identity

$$\nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla \times (\nabla \times \vec{A}) \rightarrow (8)$$

5) Problem for magnetic field.

*) The magnetic field strength of a 200 turn coil excited by a current of 2A is to be determined. The length of the Solenoid is 0.25m. The direction of the field strength is already known.

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Sol

No. of turns $N = 200$

Current flowing in the circuit $I = 2A$

Length of the Solenoid $l = 0.25m$

Magnetic field intensity $H = \frac{NI}{l} = \frac{200 \times 2}{0.25}$

$$\boxed{H = 1600 \text{ A/m}}$$

*) A ferrite material has $\mu_r = 50$, operate with sufficiently low flux densities & $B = 0.05T$ find H

$\mu_r = 50 \quad B = 0.05T$

$B = \mu H$

$B = \mu_0 \mu_r H$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{0.05}{4\pi \times 10^{-7} \times 50}$$

$$\boxed{H = 795.7747 \text{ A/m}}$$