# EC3452-ELECTROMAGNETIC FIELDS

## (Regulation 2021)

## UNIT I - INTRODUCTION

### Part A - Two Marks

**1. Define scalar field?**

A field is a system in which a particular physical function has a value at each and every point in that region. The distribution of a scalar quantity with a defined position in a space is called scalar field.

Ex: Temperature of atmosphere.

**2. Define Vector field?**

If a quantity which is specified in a region to define a field is a vector then the corresponding field is called vector field.

**3. Define scaling of a vector?**

This is nothing but, multiplication of a scalar with a vector. Such a multiplication changes the magnitude of a vector but not the direction.

**4. What are co-planar vector?**

The vectors which lie in the same plane are called co-planar vectors.

**5. Define base vectors?**

The base vectors are the unit vectors which are strictly oriented along the directions of the coordinate axes of the given coordinate system.

**6. What is a position vector?**

Consider a point $p(x, y, z)$ are Cartesian coordinate system. Then the position vector of point p is represented by the distance of point p from the origin directed from origin to point. This is also called as radius vector.

**7. Define Divergence.**

Divergence is defined as the net outward flow of the flux per unit volume over a closed incremental surface.

**8. State Divergence Theorem.**

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed that closed surface.

**9. What is physical significance of curl of a vector field?**

Curl gives rate of rotation. Curl F gives work done per unit area.

**10. What is physical significance of divergence?**

Divergence of current density gives net outflow of current per unit volume. Divergence of flux density gives net outflow per unit volume. In general, divergence of any field density gives net outflow of that field per unit volume.

11. **State the conditions for a field to be** a) solenoidal b) irrotational.
a) Divergence of the field has to be zero.

b) Curl of the field has to be zero.

**12. Define scalar and vector quantity?**

The scalar is a quantity whose value may be represented by a single real number which may be positive or negative. e.g, temperature, mass, volume, density.

A quantity which has both a magnitude and a specified direction in space is called a vector. e.g. force, velocity, displacement, acceleration.

**13. What is a unit vector? What is its function while representing a vector?**

A unit vector has a function to indicate the direction. Its magnitude is always unity, irrespective of the direction which it indicates and the coordinate system under consideration.

**14. Name 3 coordinate systems used in electromagnetic engineering?**

1) Cartesian or rectangular coordinate system.

2) Cylindrical coordinate system.

3) Spherical coordinate system.

**15. How to represent a point in a Cartesian system?**

A point in rectangular coordinate system is located by three coordinates namely x, y and z coordinates. The point can be reached by moving from origin, the distance x in x direction then the distance y in y direction and finally z in z direction.

**16. What is separation of vector?**

The distance vector is also called as separation vector. Distance vector is nothing but the length of the vector.

**17. State Distance formula?**

Distance formula give the distance between the two points representing tips of the vector.

**18. Show how a point p represented in a spherical coordinate system.**

The point p can be defined as the intersection of three surfaces in spherical coordinate system.
r - Constant which is a sphere with centre as origin?
$\theta$ – Constant which is a right circular cone with apex as origin and axis as z axis?
$\Phi$ – Constant is a plane perpendicular to xy plane.

**19. State the relationship between Cartesian and spherical system?** $x = r \sin\theta \cos\Phi$
$y = r \sin\theta \sin\Phi$
$z = r \cos\theta$
Now r can be expressed as
$x^2 + y^2 + z^2 = r^2 \sin^2\theta \cos^2\Phi + r^2 \sin^2\theta \sin^2\Phi + r^2 \cos^2\theta$
$= r^2 \sin^2\theta\,[\sin^2\Phi + \cos^2\Phi] + r^2 \cos^2\theta$
$= r^2\,[\sin^2\theta + \cos^2\theta]$
$= r^2$

**20. What are the types of integral related to electromagnetic theory?**

1. Line integral
2. Surface integral
3. Volume integral

**21. Give the types of charge distribution.**

1. Line charge
2. Point charge
3. Surface charge
4. Volume charge.

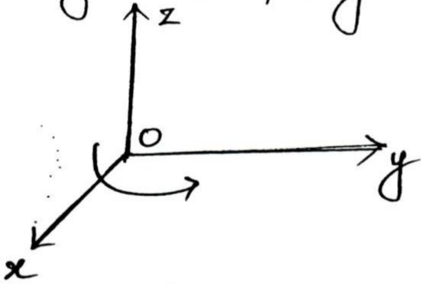
**PART-B**

**1]Explain the three coordinate systems with neat diagram.**

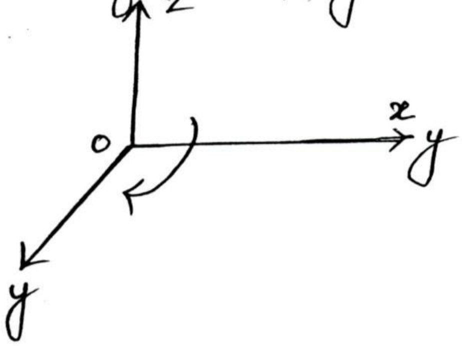
CARTESIAN COORDINATE SYSTEM / RECTANGULAR COORDINATE SYSTEM :

- The Coordinates of a Rectangular Coordinate System are $x$, $y$ and $z$.
- The Rectangular Coordinate System has three coordinate axes represented as $x$, $y$ and $z$ which are mutually right angles to each other.
- These three axes intersects at a Common point Called origin.
- There are two types,

Right handed System



Left handed System.



RANGE OF VARIABLES :

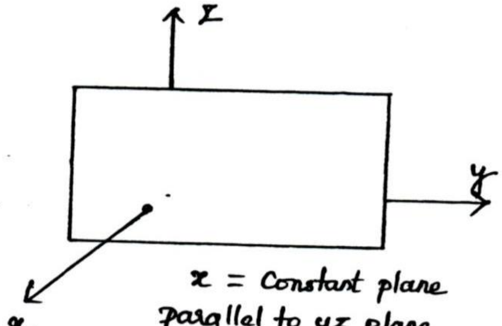
$$-\infty \le x \le \infty$$

$$-\infty \le y \le \infty$$

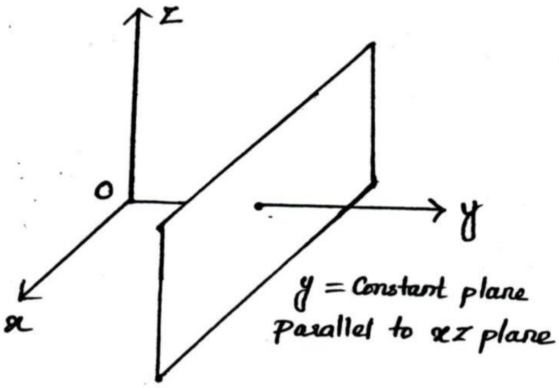
$$-\infty \le z \le \infty$$

REPRESENTATION OF A POINT :

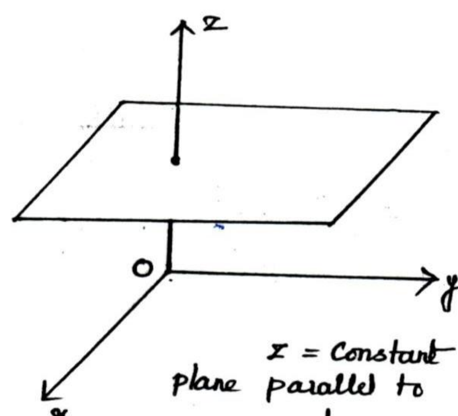
A point in Rectangular Coordinate System can be represented as the intersection of $x$ = Constant plane, $y$ = Constant plane and $z$ = Constant plane.



$x$ = Constant plane parallel to $yz$ plane



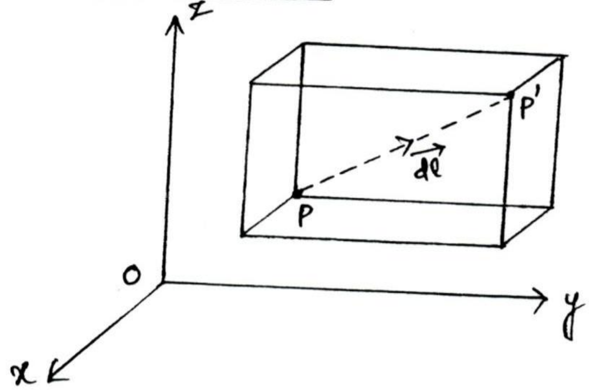
$y$ = Constant plane Parallel to $xz$ plane



$z$ = Constant plane parallel to $xy$ plane.

# DIFFERENTIAL ELEMENTS:

## (a) DIFFERENTIAL LENGTH:




$P \rightarrow (x, y, z)$

$P' \rightarrow (x+dx, y+dy, z+dz)$

- Let a point $P(x, y, z)$ in the rectangular Coordinate System.
- By increasing each coordinate by differential amount a new point $P'(x+dx, y+dy, z+dz)$ obtained.

$dx$ – Differential length in $x$ direction

$dy$ – Differential length in $y$ direction

$dz$ – Differential length in $z$ direction

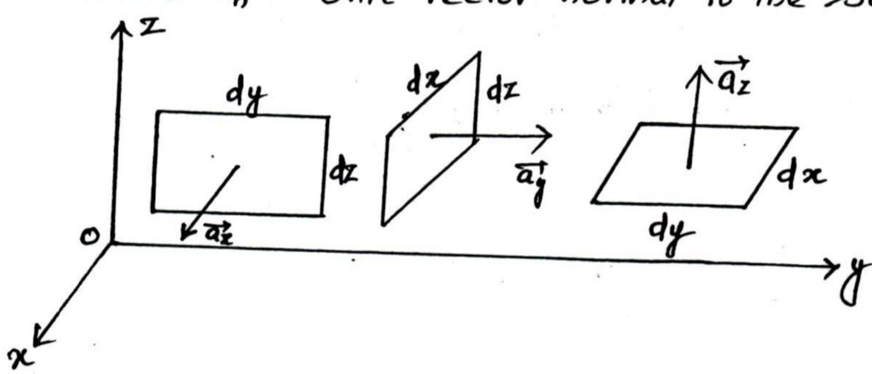
$$\left.\begin{array}{r}\text{Differential vector length/}\\ \text{Elemental vector length}\end{array}\right\} \vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$dl = |\vec{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

## (b) DIFFERENTIAL SURFACE:

$$\vec{dS} = dS\,\vec{a_n}$$

where $\vec{a_n}$ – Unit vector normal to the surface $dS$.




$$\vec{dS_x} = dy\,dz\,\vec{a_x}$$

$$\vec{dS_y} = dx\,dz\,\vec{a_y}$$

$$\vec{dS_z} = dx\,dy\,\vec{a_z}$$

## (c) DIFFERENTIAL VOLUME:

$$\left.\begin{array}{r}\text{Differential}\\ \text{Volume}\end{array}\right\} dV = dx\,dy\,dz$$

# REPRESENTATION OF A VECTOR:

The Vector in Rectangular Coordinate System can be expressed as

# CIRCULAR COORDINATE SYSTEM / CYLINDRICAL COORDINATE SYSTEM:

- Circular Coordinate System is a three dimensional version of pola system.
- The Coordinates of a Cylindrical Coordinate System are $\rho$, $\phi$
- Surfaces used to define a Cylindrical Coordinate system are,

(a) Constant $z$ plane which is parallel to $xy$ plane.

(b) A Cylinder of radius $\rho$ with $z$ axis as the axis of the C

(c) A half plane perpendicular to $xy$ plane at an angle of $\phi$ to $xz$ plane.

## RANGE OF VARIABLES:

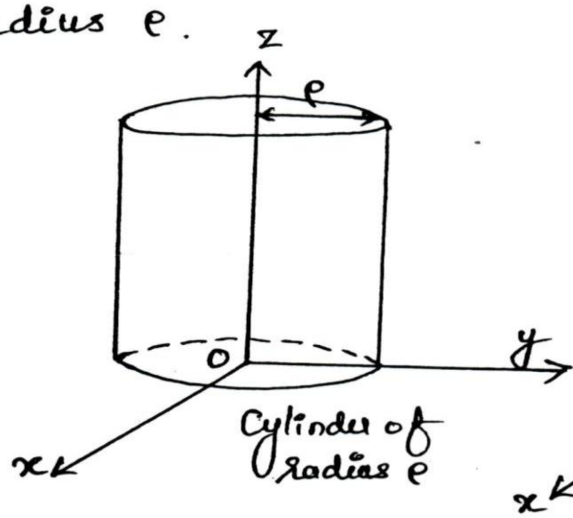
$$0 \leq \rho \leq \infty$$

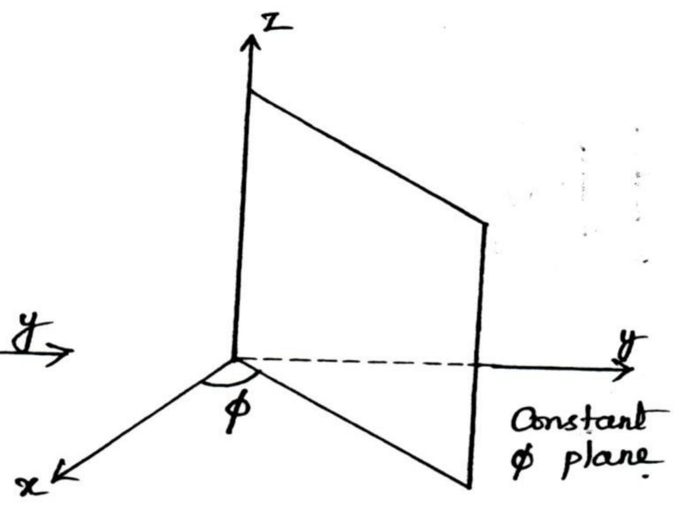
$$0 \leq \phi \leq 2\pi$$

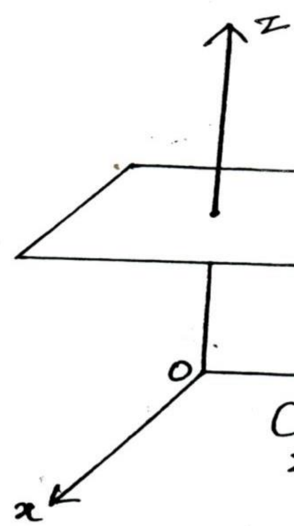
$$-\infty \leq z \leq \infty$$

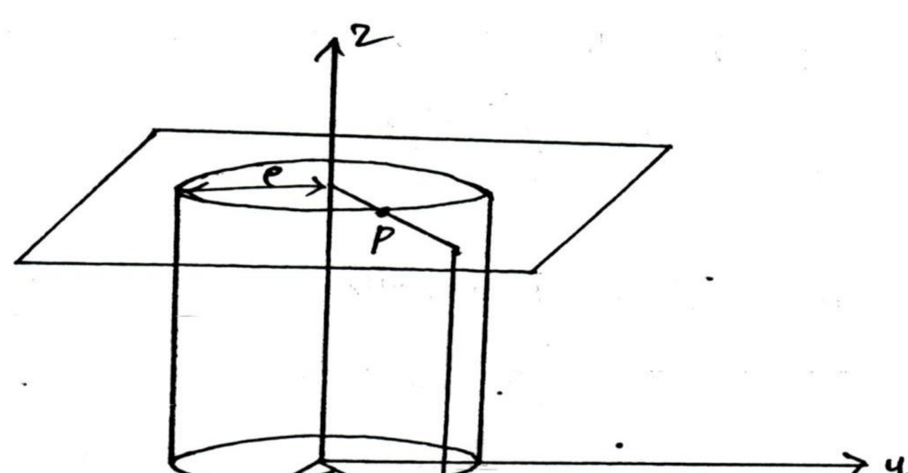
## REPRESENTATION OF A POINT:

A point in Cylindrical Coordinate System can be represen intersection of Constant $z$ plane, Constant $\phi$ plane and a Cylin radius $\rho$.



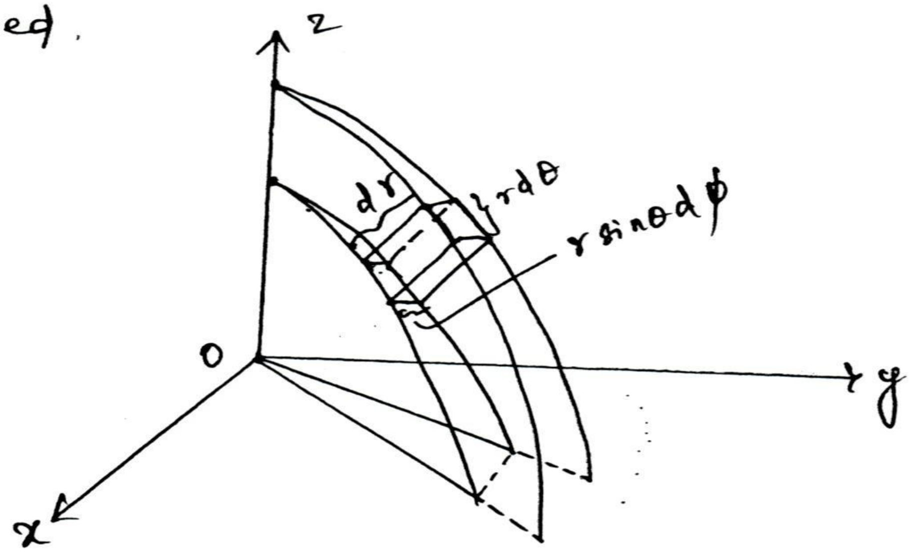



## DIFFERENTIAL ELEMENTS:

### (a) DIFFERENTIAL LENGTH:

- Let a point $P(r, \theta, \phi)$ in Spherical Coordinate System.
- By increasing each coordinate by differential amount a $P'$ obtained.




$dr$ – Differential length in $r$ direction

$r d\theta$ – Differential length in $\theta$ direction

$r \sin\theta \, d\phi$ – Differential length in $\phi$ direction.

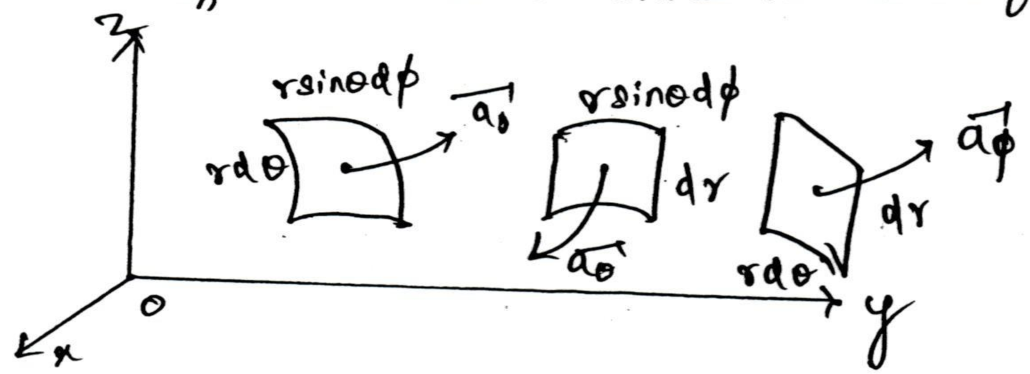
Differential Vector Length, $\vec{dl} = dr\,\vec{a_r} + r d\theta\,\vec{a_\theta} + r\sin\theta\, d\phi\,\vec{a_\phi}$

$$dl = |\vec{dl}| = \sqrt{(dr)^2 + (r d\theta)^2 + (r\sin\theta\, d\phi)^2}$$

### (b) DIFFERENTIAL SURFACE:

$$\vec{dS} = dS\,\vec{a_n}$$

where $\vec{a_n}$ – Unit Vector normal to the Surface $dS$.




$\vec{dS_r} = r^2 \sin\theta\, d$

$\vec{dS_\theta} = r\sin\theta\, d$

$\vec{dS_\phi} = r\,dr\,d\theta$

### (C) DIFFERENTIAL VOLUME:

Differential Volume, $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

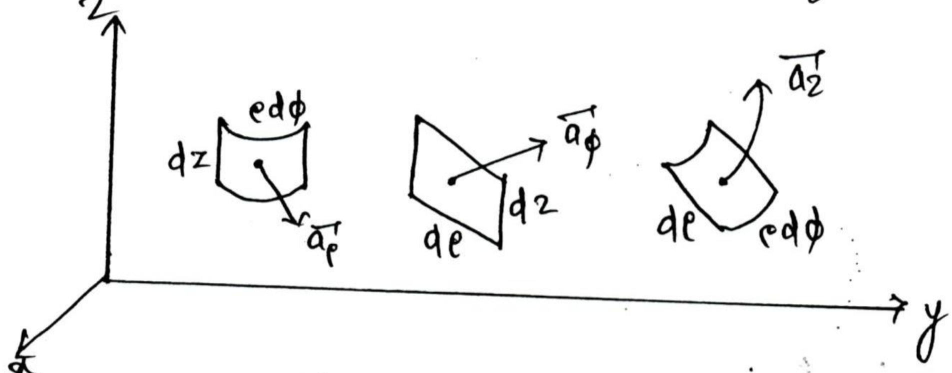
Differential Vector length, $\vec{dl} = d\rho\,\vec{a_\rho} + \rho\, d\phi\,\vec{a_\phi} + dz\,\vec{a_z}$

$$dl = |\vec{dl}| = \sqrt{(d\rho)^2 + (\rho\, d\phi)^2 + (dz)^2}$$

(b) DIFFERENTIAL SURFACE:

$$\vec{dS} = dS\,\vec{a_n}$$

where $\vec{a_n}$ - Unit Vector normal to the Surface dS.




$$\vec{dS_\rho} = \rho\, d\phi\, dz\,\vec{a_\rho}$$

$$\vec{dS_\phi} = d\rho\, dz\,\vec{a_\phi}$$

$$\vec{dS_z} = \rho\, d\rho\, d\phi\,\vec{a_z}$$

(c) DIFFERENTIAL VOLUME:

Differential Volume, $dV = \rho\, d\rho\, d\phi\, dz$.

REPRESENTATION OF A VECTOR:

The vector in Cylindrical Coordinate System can be expressed as

$$\vec{A} = A_\rho\,\vec{a_\rho} + A_\phi\,\vec{a_\phi} + A_z\,\vec{a_z}.$$

SPHERICAL COORDINATE SYSTEM:

- The Coordinates of a Spherical Coordinate System are $r$, $\theta$ and $\phi$.
- Surfaces used to define a Spherical Coordinate System are,

(a) A Sphere of radius $r$, origin as the Center of the Sphere

(b) A right circular Cone with its apex at the origin and its axis as z-axis. Its half angle is $\theta$. It rotates about z axis and $\theta$ varies from 0 to 180°.

(c) A half plane perpendicular to xy plane Containing z-axis, making an angle $\phi$ with the xz plane.

RANGE OF VARIABLES:

$$0 \le r \le \infty$$

$$0 \le \phi \le 2\pi$$

**2) problem convert cylindrical to Cartesian co ordinate system's**

PROBLEM:

① Give the Cylindrical Coordinates of the point whose Cartesian Coordinates are $x = 3$, $y = 4$ and $z = 5$ units.

Given: $x = 3$, $y = 4$, $z = 5$

Solution:

$$\rho = \sqrt{x^2 + y^2} = \sqrt{9+16} = \sqrt{25}$$
$$\boxed{\rho = 5}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$
$$\boxed{\phi = 53.13^\circ}$$

$$z = z$$
$$\boxed{z = 5}$$

The cylindrical Coordinates are $\rho = 5$, $\phi = 53.13^\circ$, $z = 5$

② Give the Cartesian Coordinates of the point whose Cylindrical Coordinates are $\rho = 2$, $\phi = 45^\circ$ and $z = -1$

Given: $\rho = 2$, $\phi = 45^\circ$, $z = -1$

Solution;

$$x = \rho \cos\phi = 2 \cos 45^\circ$$
$$\boxed{x = 0.707}$$

$$y = \rho \sin\phi = 2 \sin 45^\circ$$
$$\boxed{y = 0.707}$$

$$z = z$$
$$\boxed{z = -1}$$

The Cartesian Coordinates are $x = 0.707$, $y = 0.707$, $z = -1$.

③ Give the Spherical Coordinates of the point whose Cartesian Coordinates are $x = -1$, $y = 3$ and $z = 5$

Given: $x = -1$, $y = 3$, $z = 5$

Solution:

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1+9+25}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{\ }\right)$$

$$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

**3) problem convert cartesian to spherical co ordinate system's**

(4) Give the Cartesian Coordinates of the point whose Spherical Coordinates are $r = 3$, $\theta = 60^\circ$ and $\phi = 30^\circ$

Given: $r = 3$, $\theta = 60^\circ$, $\phi = 30^\circ$

Solution:

$x = r \sin\theta \cos\phi$

$= 3 \sin 60^\circ \cos 30^\circ$

$= 3 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{9}{4}$

$\boxed{x = 2.25}$

$y = r \sin\theta \sin\phi$

$= 3 \sin 60^\circ \sin 30^\circ$

$= 3 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\boxed{y = \quad}$

$z = r \cos\theta$

$= 3 \cos 60^\circ$

$= \frac{3}{2}$

$\boxed{z = 1.5}$

The Cartesian Coordinates are $x = 2.25$, $y = \quad$, $z = 1.5$.

(5) Given Points $A(x = 2, y = 3, z = -1)$ and $B(\rho = 4, \phi = -50^\circ, z = 2)$. Find the distance from A to B.

Given: $A(x = 2, y = 3, z = -1)$

$B(\rho = 4, \phi = -50^\circ, z = 2)$

$x = \rho \cos\phi$

$= 4 \cos(-50^\circ)$

$\boxed{x = 2.571}$

$y = \rho \sin\phi$

$= 4 \sin(-50^\circ)$

$\boxed{y = -3.064}$

$z = z$

$\boxed{z = 2}$

$B(\rho = 4, \phi = -50^\circ, z = 2) = B(x = 2.571, y = -3.064, z = 2)$

$$\left.\begin{matrix}\text{Distance from}\\ \text{A to B}\end{matrix}\right\} = \sqrt{(2.571 - 2)^2 + (-3.064 - 3)^2 + (2+1)^2}$$

$$= \sqrt{46.098}$$

$$\boxed{d = 6.789}$$

**4) Determine the divergence Theorem.**

PROBLEM:



① Determine the divergence of the vector fields.

(i) $\vec{P} = x^2yz\,\vec{a}_x + xz\,\vec{a}_z$

(ii) $\vec{Q} = \rho\sin\phi\,\vec{a}_\rho + \rho^2 z\,\vec{a}_\phi + z\cos\phi\,\vec{a}_z$.

(iii) $\vec{T} = \frac{1}{r^2}\cos\theta\,\vec{a}_r + r\sin\theta\cos\phi\,\vec{a}_\theta + \cos\theta\,\vec{a}_\phi$

Solution:

for Cartesian:

$$\nabla\cdot\vec{P} = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$$

Given: $P_x = x^2yz$, $P_y = 0$, $P_z = xz$

$$\nabla\cdot\vec{P} = \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$\boxed{\nabla\cdot\vec{P} = 2xyz + x}$$

for Cylindrical:

$$\nabla\cdot\vec{Q} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial Q_\phi}{\partial\phi} + \frac{\partial Q_z}{\partial z}$$

Given: $Q_\rho = \rho\sin\phi$, $Q_\phi = \rho^2 z$, $Q_z = z\cos\phi$

$$\nabla\cdot\vec{Q} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2\sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho^2 z) + \frac{\partial}{\partial z}(z\cos\phi)$$

$$= \frac{1}{\rho}(2\rho\sin\phi) + \frac{1}{\rho}(0) + \cos\phi$$

$$\boxed{\nabla\cdot\vec{Q} = 2\sin\phi + \cos\phi}$$

for Spherical:

$$\nabla\cdot\vec{T} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(T_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial T_\phi}{\partial\phi}$$

Given: $T_r = \frac{1}{r^2}\cos\theta$, $T_\theta = r\sin\theta\cos\phi$, $T_\phi = \cos\theta$

$$\nabla\cdot\vec{T} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\cancel{r^2}\times\frac{1}{\cancel{r^2}}\cos\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\cos\theta)$$

**5) problem convert cartesian to spherical co ordinate system's**

(2) Given that $\vec{D} = \frac{10\rho^3}{4}\vec{a_\rho}$ in Cylindrical Coordinate System. Evaluate both sides of divergence theorem for the volume enclosed with $\rho = 2$ and $z = 0$ & $z = 10$.

Given:

$$\vec{D} = \frac{10\rho^3}{4}\vec{a_\rho}$$

Solution:

By divergence theorem

$$\iiint_V \nabla\cdot\vec{D}\, dv = \iint_S \vec{D}\cdot\vec{ds}$$

LHS: $\iiint_V \nabla\cdot\vec{D}\, dv$

$$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho \times \frac{10\rho^3}{4}\right) = \frac{1}{\rho}\times\frac{10}{4}\times 4\rho^3$$

$$\nabla\cdot\vec{D} = 10\rho^2$$

$$\iiint_V \nabla\cdot\vec{D}\, dv = \int_{z=0}^{10}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{2} 10\rho^2 \times \rho\, d\rho\, d\phi\, dz \qquad \because dv = \rho\, d\rho\, d\phi\, dz$$

$$= 10\int_0^{10}\int_0^{2\pi}\left[\frac{\rho^4}{4}\right]_0^2 d\phi\, dz$$

$$= 10\times\frac{16}{4}\int_0^{10}\left[\phi\right]_0^{2\pi} dz$$

$$= 40\times 2\pi\left[z\right]_0^{10}$$

$$= 40\times 2\pi\times 10$$

$$\iiint_V \nabla\cdot\vec{D}\, dv = 800\pi \quad \text{——————} \quad (1)$$

RHS: $\oiint \vec{D}\cdot\vec{ds}$

$$\oiint \vec{D}\cdot\vec{ds} = \iint_{\rho=2} \vec{D}\cdot ds_\rho\vec{a_\rho} + \iint \vec{D}\cdot ds_z\vec{a_z}$$

Given:

$$\vec{D} = \frac{10\rho^3}{4}\vec{a_\rho}$$

Solution:

By divergence theorem

$$\iiint_V \nabla\cdot\vec{D}\,dV = \iint_S \vec{D}\cdot\vec{ds}$$

LHS: $\iiint_V \nabla\cdot\vec{D}\,dV$

$$\nabla\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\times\frac{10\rho^3}{4}\right) = \frac{1}{\rho}\times\frac{10}{4}\times 4\rho^3$$

$$\nabla\cdot\vec{D} = 10\rho^2$$

$$\iiint_V \nabla\cdot\vec{D}\,dV = \int_{z=0}^{10}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{2} 10\rho^2 \times \rho\,d\rho\,d\phi\,dz \qquad \because dV =$$

$$= 10\int_0^{10}\int_0^{2\pi}\left[\frac{\rho^4}{4}\right]_0^2 d\phi\,dz$$

$$= 10\times\frac{16}{4}\int_0^{10}[\phi]_0^{2\pi}dz$$

$$= 40\times 2\pi\,[z]_0^{10}$$

$$= 40\times 2\pi\times 10$$

$$\iiint_V \nabla\cdot\vec{D}\,dV = 800\pi \longrightarrow (1)$$

RHS: $\oiint \vec{D}\cdot\vec{ds}$

$$\oiint \vec{D}\cdot\vec{ds} = \iint_{\rho=2}\vec{D}\cdot ds_\rho\vec{a_\rho} + \iint \vec{D}\cdot ds_z\vec{a_z}$$

$$= \iint_{\rho=2}\frac{10\rho^3}{4}\times\rho\,d\phi\,dz + 0$$

$$= \frac{10}{4}(2)^4\int_0^{10}\int_0^{2\pi}d\phi\,dz$$